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Experimental characterisation of the motion of an inverted pendulum

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Abstract: In this paper, we present a home-made experimental set-up to study the falling movement of an inverted pendulum. The experimental set-up allows preparing a laboratory session for first year Physics or Engineering students. This set-up has been used in the Bachelor's Degree in Mechanical Engineering at the School of Design Engineering of the Universitat Politècnica de València. The experimental data are fitted to the theoretical equation of motion, obtaining a very good agreement between experiment and theory. In addition, direct measurement of the parameters involved in the equations was carried out, showing a very good agreement with the calculated parameters.

Keywords: Inverted pendulum; electro-optical sensor; Physics; Energy conservation

Introduction

An inverted pendulum is a pendulum that has the centre of mass above its pivot point. It can be make with a rigid rod which rotates in a vertical plane: Stephenson (1908), Phelps & Hunter (1965), Blitzer (1965), Friedman (1982) and Douvropoulos (2012). The inverted pendulum has the remarkable property to be stable in a vertical position when the supporting point oscillates vertically above a certain frequency: Corben & Stehle (1994) and Semenov *el al*. (2013). This system has been used for teaching purpose in nonlinear dynamics and control theory engineering: see for instance Hovland (2008), Magana & Holzapfel (1998), Zhao & Spong (2001), among many other references that can be found in the literature.

Analytical and approximate solutions for the differential equation of the non-linear pendulum can be found in the literature (see for instance Matthews *el al*. (2005) and references therein).

In this paper, we present a home-made experimental set-up to study the falling movement of an inverted pendulum. The theoretical analysis of the system is based in the law of conservation of mechanical energy, that is an important topic of physics in the first year of many Engineering Bachelor's Degrees. In particular, this set-up has been used in a laboratory session in the subject of physics in the Bachelor's Degree in Mechanical Engineering at the School of Design Engineering of the Universitat Politècnica de València.

Material and Methods

The experimental set up is shown in Figure 1a. It consists of two rigid rods (1) than can rotate around an articulated joint (2). Between the two rods, there is a half circle protractor (3) marked in degrees, with 7 holes, and the set-up also has an electro-optical sensor (4) for detecting the pass through the holes. The holes have an angular size of 2 degrees, and the distance between holes is gradually increased. The first hole is placed at 7 degrees due to the size of the detection system.

The electro-optical sensor is composed of a light-emitting diode (led) and a receiver phototransistor, placed one in front of the other. When the system is passing in front of the holes, the light radiation from the led is received by the phototransistor producing a current which is measured in an oscilloscope connected to the system.

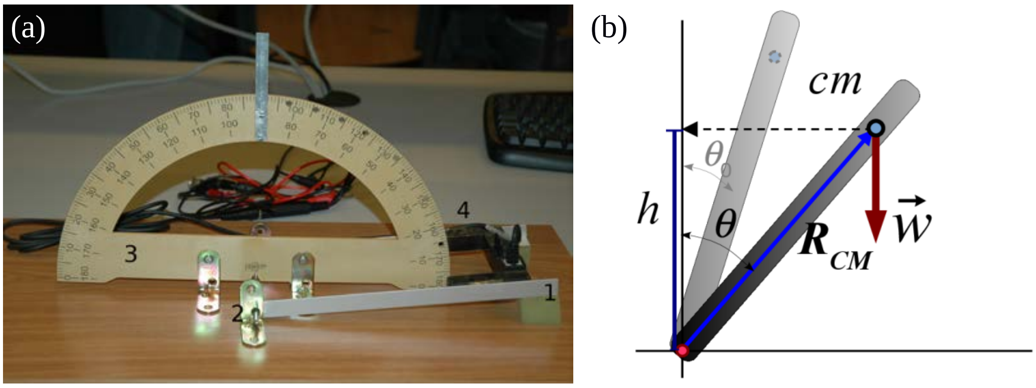

Figure 1. (a) Experimental set-up for measuring the free fall of an inverted pendulum. (b) Schematic representation of the set-up for measuring the free fall of an inverted pendulum.

It should be important to note that the cost of the proposed system is very cheap, except for the oscilloscope, which is used for data measuring, although this should not be an implementation difficulty as this is very common general equipment in Physics and Electricity laboratories.

The system is schematically represented in Figure 1b, where θ is the angle between the vertical line and the rod. The represented point *cm* is the position of the Centre of Mass of the system, and $\vec{w}$ is the weight of the rod.

The potential energy of the system when the rod makes an angle θ with the vertical line is given by:

$$U = mgh = mgR_{CM}cos\theta \quad (1)$$

where m is the total mass of the system, g is the gravitational acceleration, h is the height of the centre of mass and R_{CM} is the distance from the rotation point to the system centre of mass.

The kinetic energy is given by,

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}mR_g^2\omega^2 \quad (2)$$

where in this last expression ω is the angular velocity, I is the moment of inertia, and the radius of gyration, R_g, has been introduced:

$$R_g = \sqrt{\frac{I}{m}} \quad (3)$$

The system is dropped from an initial angle θ_0, without initial velocity. From the energy conservation of mechanical energy, neglecting the loss of energy due to frictional forces, the kinetical initial energy at the initial point has to be equal to the total energy when the rod makes an angle θ with the vertical line:

$$mgR_{CM}cos\theta_0 = mgR_{CM}cos\theta + \frac{1}{2}mR_g^2\omega^2 \quad (4)$$

Then, we can obtain the square of the angular velocity:

$$\omega^2 = 2K^2(cos\theta_0 - cos\theta) \quad (5)$$

where, for simplicity in the expression, we have introduce the constant *K*:

$$K^2 = \frac{gR_{CM}}{R_g^2} \quad (6)$$

Then, the students should measure the angular velocity as a function of the angular position. The angular velocity can be approximately calculated as the size of the hole (2 degrees = 0.0349 rad in the proposed set-up) divided by the time it takes to pass over the hole, that it is measured with the oscilloscope:

$$\omega \simeq \frac{\Delta\theta}{\Delta t} \quad (7)$$

Within this approximation, the students should graphically represented the square of the angular velocity *vs* the cosine of the angle position, and a straight line should be obtained. From this data, performing a linear fit, the slope of the straight line is equal to $2K^2$. From the y-intercept of the line, the value of the initial angle can be calculated.

Results and Discussion

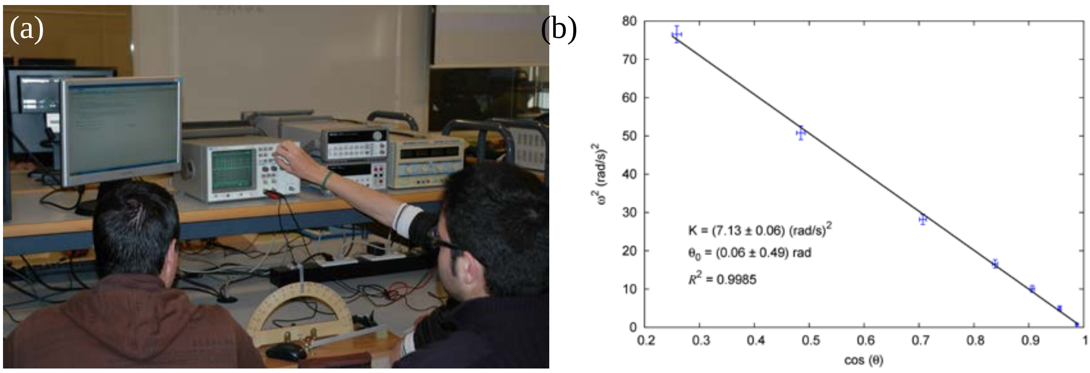

Figure 2. (a) A group of students performing the measurement. (b) Angular velocity squared *vs* cos θ; crosses represent data points with error bars, and solid line the linear fit.

In the picture of Figure 2a, it can be seen a group of students performing the measurement in the oscilloscope with the proposed set-up. In Figure 2b, the square of the angular velocity as a function of angular position is represented. A linear fit has also been carried out in order to obtain the slope = – (101.7 ± 1.8) $(rad/s)^2$ and the y-intercept point = (101.5 ± 1.4) $(rad/s)^2$. From these results, the value of *K* and θ_0 shown in Figure 2 are obtained.

The results obtained for the value for the fit parameters are very good, with relative errors smaller than 2%, calculated as the ration between the absolute error and the result of measurement. This result is quite remarkable, because of the simplicity of the experimental set-up together with the approximations done in calculations. Some points in the graph of Figure 2b vary slightly from the trend line, which can be attributed to the approximation for the value of the angular velocity.

In order to compare the results obtained, direct measurement of the parameters can be performed by the students. For the angle θ_0, the initial position of the rod can be directly measured with the half circle protractor, obtaining the value θ_0 = (0.096 ± 0.009) rad. The agreement is not very good. The reason of this discrepancy for the angle θ_0 is due to the fact that we have represented the value of cos θ, that for the case of θ_0 is very close to 1. Then, when the arccos of this number is calculated and the propagation of error is performed, a large value of the error is obtained.

On the other hand, the fundamental equation is the same for the pendulum in inverse position as for the normal position (Matthews et al., 2005). Therefore, students can place the moving part in the normal pendulum position, and make it oscillate over its equilibrium position. The angular frequency of the oscillation is equal to the *K* constant. From the oscillation period for small oscillations, they can find the value of the *K* constant (7.09 ± 0.06) rad/s. In this case, we have a very good agreement between the obtained value and the direct measurement of the *K* constant, with a discrepancy smaller than 1%.

Conclusions

In this paper we have presented a home-made experimental set-up to measure the motion of an inverted pendulum fall. This experimental set-up has been implemented as a laboratory session in the subject of physics in the first year of the Bachelor's Degree in Mechanical Engineering at the School of Design Engineering of the Universitat Politècnica de València.

In spite of the approximation done to calculate the angular velocity, a very good agreement between the fitted parameters and direct measurement of these parameters has been found.

After the lab session, the students have to present a laboratory report. From the results of this laboratory report, we have checked that most of the students reach the planned learning objectives. During the lab session, we have checked that the students are motivated by this experiment. In particular, the students are surprised about the good results in spite of the simplicity of the experiment. In any case, to complete this study, we are planning to take a survey to the students, in order to have feedback, and to check their motivation.

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References

Blitzer, L. (1965). Inverted Pendulum. *American Journal of Physics*, *33*(12), 1076–1077. doi:10.1119/1.1971158

Corben, H. C., & Stehle, P. (1994). *Classical Mechanics*. New York: Yohn Wiley & Sons.

Douvropoulos, T. G. (2012). Simple analytic formula for the period of the nonlinear pendulum via the Struve function: connection to acoustical impedance matching. *European Journal of Physics*, *33*(1), 207–217. doi:10.1088/0143-0807/33/1/018

Friedman, M. H. (1982). The inverted pendulum: A mechanical analog of the quadrupole mass filter. *American Journal of Physics*, *50*(10), 924–930. doi:10.1119/1.13015

Hovland, G. (2008). Evaluation of an Online Inverted Pendulum Control Experiment. *IEEE Transactions on Education*, *51*(1), 114–122. doi: 10.1109/TE.2007.906612

Magana, M.E., & Holzapfel, F. (1998). Fuzzy-logic control of an inverted pendulum with vision feedback. *IEEE Transactions on Education*, *41*(2), 165–170. doi: 10.1109/13.669727

Matthews, M. R., Gauld, C. F., & Stinner, A. (2005). *The Pendulum: Scientific, Historical, Philosophical and Educational Perspectives*. The Netherlands: Springer Science & Business Media.

Phelps, F. M., & Hunter, J. H. (1965). An Analytical Solution of the Inverted Pendulum. *American Journal of Physics*, *33*(4), 285–295. doi:10.1119/1.1971474

Semenov, M. E., Shevlyakova, D. V., & Meleshenko, P. A. (2013). Inverted pendulum under hysteretic control: stability zones and periodic solutions. *Nonlinear Dynamics*, *75*(1-2), 247–256. doi:10.1007/s11071-013-1062-x

Stephenson, A. (1908). On induced stability. *Philosophical Magazine Series 6*, *15*(86), 233–236. doi:10.1080/14786440809463763

Zhao, J. & Spong, M.W. (2001). Hybrid control for global stabilization of the cart–pendulum system, *Automatica*, *37*(12), 1941–1951. doi:10.1016/S0005-1098(01)00164-9.

