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cl-Supercontinuous Functions

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Abstract. Basic properties of cl-supercontinuity, a strong variant of continuity, due to Reilly and Vamanamurthy [Indian J. Pure Appl. Math., 14 (1983), 767–772], who call such maps clopen continuous, are studied. Sufficient conditions on domain or range for a continuous function to be cl-supercontinuous are observed. Direct and inverse transfer of certain topological properties under cl-supercontinuous functions are studied and existence or nonexistence of certain cl-supercontinuous function with specified domain or range is outlined.

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1. Introduction

Strong variants of continuity are of considerable significance and arise in many branches of mathematics including topology, complex analysis and functional analysis. Reilly and Vamanamurthy [12] call a function $f : X \to Y$ clopen continuous if for each open set V containing $f(x)$ there exists a clopen (closed and open) set U containing x such that $f(U) \subset V$. In this paper we elaborate on the properties of these mappings introduced by Reilly and Vamanamurthy. However, in the topological folklore the phrase "clopen map" is used for the functions which map clopen sets to open sets. So in this paper we rename "clopen continuous maps" as cl-supercontinuous functions which appears to be a better nomenclature, since it is a strong form of supercontinuity introduced by Munshi and Basan [9].

The class of cl-supercontinuous functions strictly contains the class of perfectly continuous functions of Noiri [11] which in turn properly include all strongly continuous functions of Levine [8]. Furthermore, the class of cl-supercontinuous functions is properly contained in the class of z-supercontinuous functions [3] which in its turn is contained in the class of supercontinuous functions [9].

In Section 2, several characterizations of cl-supercontinuity are obtained and it is shown that cl-supercontinuity is preserved under restrictions, compositions, products, and passage to the graph function. The notions of cl-quotient topology and cl-quotient space are introduced in Section 3. Section 4 is devoted to the study of the behavior of separation axioms under cl-supercontinuous functions. In Section 5 we conclude with alternative proofs of certain results of preceding sections. Lastly, we mention some possible application of cl-supercontinuity to topology and analysis.

2. Basic Properties of cl-Supercontinuous Functions

Definition 2.1. *A set G in a topological space X is said to be **cl-open** if for each $x \in G$, there exist a clopen set H such that $x \in H \subseteq G$, equivalently G is the union of clopen sets. The complement of a cl-open set will be referred to as **cl-closed** set.*

Theorem 2.2. *For a function $f : X \to Y$, the following statements are equivalent.*

(a) *f is cl-supercontinuous.*
(b) *Inverse image of every open subset of Y is a cl-open in X.*
(c) *Inverse image of every closed subset of Y is a cl-closed in X.*

Proof of Theorem 2.2 is routine and hence omitted.

Remark 2.3. If either of the spaces X and Y is zero-dimensional, then any continuous function from X to Y is cl-supercontinuous.

Definition 2.4. *Let X be a topological space and let $A \subset X$. A point $x \in X$ is said to be a **cl-adherent** of A if every clopen set containing x intersects A. Let $[A]_{cl}$ denote the set of all cl-adherent points of A. Then the set A is cl-closed if and only if $[A]_{cl} = A$.*

Theorem 2.5. *For a function $f : X \to Y$ the following statements are equivalent.*

(a) *f is cl-supercontinuous.*
(b) *$f([A]_{cl}) \subset \overline{f(A)}$ for every set $A \subset X$.*
(c) *$[f^{-1}(B)]_{cl} \subset f^{-1}(\bar{B})$ for every $B \subset Y$.*

Proof. $(a) \Rightarrow (b)$. Since $\overline{f(A)}$ is closed in Y, by Theorem 2.2 $f^{-1}(\overline{f(A)})$ is a cl-closed in X. Again, since $A \subset f^{-1}(\overline{f(A)})$, $[A]_{cl} \subset [f^{-1}(\overline{f(A)})]_{cl} = f^{-1}(\overline{f(A)})$ and so $f([A]_{cl}) \subset f(f^{-1}(\overline{f(A)}) \subset \overline{f(A)}$.
$(b) \Rightarrow (c)$. Let $B \subset Y$. Then by (b), $f([f^{-1}(B)]_{cl}) \subset \overline{f(f^{-1}(B))} \subset \bar{B}$ and so it follows that $[f^{-1}(B)]_{cl} \subset f^{-1}(\bar{B})$.
$(c) \Rightarrow (a)$. Let F be any closed set in Y. Then $[f^{-1}(F)]_{cl} \subset f^{-1}(\bar{F}) = f^{-1}(F)$. Again, since $f^{-1}(F) \subset (\overline{f^{-1}(F)}) \subset [f^{-1}(F)]_{cl}$, $f^{-1}(F) = [f^{-1}(F)]_{cl}$ which in turn implies that $f^{-1}(F)$ is cl-closed and so in view of Theorem 2.2 f is cl-supercontinuous. □

Definition 2.6. *A filter base $\digamma$ is said to **cl-converge** to a point x written as $\digamma \xrightarrow{cl} x$ if every clopen set containing x contains a member of $\digamma$.*

Theorem 2.7. *A function $f : X \to Y$ is cl-supercontinuous if and only if for each $x \in X$ and each filter base $\digamma$ that cl-converges to x, $f(\digamma) \to f(x)$.*

Proof. Assume that f is cl-supercontinuous and let $\digamma \xrightarrow{cl} x$. Let W be an open set containing $f(x)$. Then $x \in f^{-1}(W)$ and $f^{-1}(W)$ is cl-open. Let H be a clopen set such that $x \in H \subset f^{-1}(W)$ and so $f(H) \subset W$. Since $\digamma \xrightarrow{cl} x$, there exists a $U \in \digamma$ such that $U \subset H$ and hence $f(U) \subset f(H) \subset W$. Thus $f(\digamma) \to f(x)$. □

Conversely, let W be an open subset of Y containing $f(x)$. Now the filter base Nx consisting of all clopen sets containing x cl-converges to x and so by hypothesis $f(Nx) \to f(x)$. Hence there exists a member $f(N)$ of $f(Nx)$ such that $f(N) \subset W$. Since $N \in Nx$, N is an clopen set containing x. Thus f is cl-supercontinuous.
It is routine to verify that cl-supercontinuity is invariant under restriction and composition of functions and enlargement of range. Moreover, the composition is cl-supercontinuous whenever $f : X \to Y$ is cl-supercontinuous and $g : Y \to Z$ is continuous.

Remark 2.8. *In general cl-supercontinuity of $g \circ f$ need not imply even continuity of f. For example, let X be the real line with cofinite topology, $Y = \{0, 1\}$ be the two point Sierpinski space [16] and let $f : X \to Y$ be defined by*

$$f(x) = \begin{cases} 0, & \textit{if } x \textit{ is irrational} \\ 1, & \textit{if } x \textit{ is rational} \end{cases}$$

Let $Z = \{0, 1\}$ be endowed with the indiscrete topology and let $g : Y \to Z$ be the identity map. Then $g \circ f$ and g are cl-supercontinuous, however, f is not continuous.

Definition 2.9. *A function $f : X \to Y$ is said to be **cl-open** (**cl-closed**) if $f(A)$ is open (closed) in Y for every clopen set A in X.*

Theorem 2.10. *Let $f : X \to Y$ be a cl-open, cl-supercontinuous surjection and $g : Y \to Z$ be any function. Then $g \circ f$ is cl-supercontinuous if and only if g is continuous.*

Theorem 2.11. *Let $f : X \to Y$ be any function. If $\{U_\alpha : \alpha \in \Delta\}$ is a cl-open cover of X and for each α, $f_\alpha = f \,|\, U_\alpha : U_\alpha \to Y$ is cl-supercontinuous, then f is cl-supercontinuous.*

Proof. Let V be a clopen subset of Y. Then $f^{-1}(V) = \cup\{f_\alpha^{-1}(V) : \alpha \in \Delta\}$ and since each f_α is cl-supercontinuous, each $f_\alpha^{-1}(V)$ is cl-open in U_α and hence in X. Thus $f^{-1}(V)$ being the union of cl-open sets is cl-open. □

Theorem 2.12. *Let $\{f_\alpha : X \to X_\alpha \,|\, \alpha \in \Lambda\}$ be a family of functions and let $f : X \to \prod_{\alpha\in\Lambda} X_\alpha$ be defined by $f(x) = (f_\alpha(x))$. Then f is cl-supercontinuous if and only if each $f_\alpha : X \to X_\alpha$ is cl-supercontinuous.*

Proof. Let $f : X \to \prod_{\alpha\in\Lambda} X_\alpha$ be cl-supercontinuous. Then the composition $f_\alpha = p_\alpha \circ f$, where p_α denotes the projection of $\prod_{\alpha\in\Lambda} X_\alpha$ onto αth-coordinate space X_α, is cl-supercontinuous for each α. □

Conversely, suppose that each $f_\alpha : X \to X_\alpha$ is cl-supercontinuous. To show that the function f is cl-supercontinuous, in view of Theorem 2.2 it is sufficient to show that $f^{-1}(U)$ is cl-open for each open set U in the product space $\prod_{\alpha\in\Lambda} X_\alpha$. Since the finite intersections and arbitrary unions of cl-open sets is cl-open, it suffices to prove that f^{-1} (S) is cl-open for every subbasic open set S in the product space $\prod_{\alpha\in\Lambda} X_\alpha$. Let $U_\beta \times \prod_{\alpha\neq\beta} X_\alpha$ be a subbasic open set in $\prod_{\alpha\in\Lambda} X_\alpha$. Then $f^{-1}(U_\beta \times \prod_{\alpha\neq\beta} X_\alpha) = f^{-1}(p_\beta^{-1}(U_\beta)) = f_\beta^{-1}(U_\beta)$ is cl-open in X. Hence f is cl-supercontinuous.

Theorem 2.13. *For each $\alpha \in \Delta$, let $f_\alpha : X_\alpha \to Y_\alpha$ be a mapping and let $f : \prod X_\alpha \to \prod Y_\alpha$ be a mapping defined by $f((x_\alpha)) = (f_\alpha(x_\alpha))$ for each (x_α) in $\prod X_\alpha$. Then f is cl-supercontinuous if and only if f_α is cl-supercontinuous for each $\alpha \in \Delta$.*

Proof. Let $f : \prod X_\alpha \to \prod Y_\alpha$ be cl-supercontinuous. Let V_β be an open subset of Y_β. Then $V_\beta \times (\prod_{\alpha\neq\beta} Y_\alpha)$ is a subbasic open subset of the product space $\prod Y_\alpha$. Since f is cl-supercontinuous, $f^{-1}(V_\beta \times (\prod_{\alpha\neq\beta} Y_\alpha)) = f_\beta^{-1}(V_\beta) \times (\prod_{\alpha\neq\beta} X_\alpha)$ is cl-open in $\prod X_\alpha$. Consequently, $f_\beta^{-1}(V_\beta)$ is a cl-open set in X_β and hence f_β is cl-supercontinuous. □

Conversely, suppose that each $f_\alpha : X_\alpha \to Y_\alpha$ is cl-supercontinuous. Let $V = V_\beta \times (\prod_{\alpha\neq\beta} Y_\alpha)$ be a subbasic open set in $\prod Y_\alpha$. Since each f_α is cl-supercontinuous and since $f^{-1}(V) = f^{-1}(V_\beta \times (\prod_{\alpha\neq\beta} Y_\alpha)) = f_\beta^{-1}(V_\beta) \times (\prod_{\alpha\neq\beta} X_\alpha)$, $f^{-1}(V)$ is cl-open, and so f is cl-supercontinuous.

Theorem 2.14. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, be the graph function. Then g is cl-supercontinuous if and only if f is cl-supercontinuous and X is zero-dimensional.*

Proof. To prove necessity, suppose that g is cl-supercontinuous. Then the composition $f = p_y \circ g$ is cl-supercontinuous, where p_y is the projection from $X \times Y$ onto Y. Let U be any open set in X and let $x \in U$. Then $U \times Y$ is an open set containing $g(x)$. Since g is cl-supercontinuous, there exists a clopen set W containing x such that $g(W) \subset U \times Y$. Thus $x \in W \subset U$, which shows that U is a cl-open and so X is zero-dimensional. □

To prove sufficiency, let $x \in X$ and let W be an open set containing $g(x)$. There exist open sets $U \subset X$ and $V \subset Y$ such that $(x, f(x)) \in U \times V \subset W$. Since X is zero-dimensional, there exists a clopen set G_1 in X containing x

such that $x \in G_1 \subset U$. Since f is cl-supercontinuous, there exists a clopen set G_2 in X containing x such that $f(G_2) \subset V$. Let $G = G_1 \cap G_2$. Then G is a clopen set containing x and $g(G) \subset U \times V \subset W$, which implies that g is cl-supercontinuous.

Definition 2.15. *A function $f : X \to Y$ is said to be* ***slightly continuous*** [1] *if $f^{-1}(A)$ is open in X for every clopen set A in Y.*

Lemma 2.16. *For a function $f : X \to Y$, the following statements are equivalent.*

(a) *f is slightly continuous.*
(b) *$f(\bar{A}) \subseteq [f(A)]_{cl}$ for all $A \subseteq X$.*
(c) *$\overline{(f^{-1}(B))} \subseteq f^{-1}([B]_{cl})$ for all $B \subseteq Y$.*
(d) *Inverse image of every cl-closed set is closed.*
(e) *Inverse image of every cl-open set is open.*

Proof. $(a) \Rightarrow (b)$: Let $y \in f(\bar{A})$. Choose $x \in \bar{A}$ such that $f(x) = y$. Let V be a clopen set containing y. Since f is slightly continuous, $f^{-1}(V)$ is an open set containing x. This gives $f^{-1}(V) \cap A \neq \phi$ which in turn implies that $V \cap f(A) \neq \phi$ and consequently $y \in [f(A)]_{cl}$. Hence $f(\bar{A}) \subset [f(A)]_{cl}$.
$(b) \Rightarrow (c)$: Let B be any subset of Y. Then $f(\overline{f^{-1}(B)}) \subseteq [f(f^{-1}(B))]_{cl}$ and consequently $\overline{(f^{-1}(B))} \subseteq f^{-1}([B]_{cl})$.
$(c) \Rightarrow (d)$: Since a set A is cl-closed if and only if $A = [A]_{cl}$, therefore the implication $(c) \Rightarrow (d)$ is obvious.
$(d) \Rightarrow (e)$: Obvious.
$(e) \Rightarrow (a)$: This is immediate since every clopen set is cl-open and since a function is slightly continuous if and only if the inverse image of every clopen set is open. □

Theorem 2.17. *Let X, Y and Z be topological spaces and let the function $f : X \to Y$ be slightly continuous and $g : Y \to Z$ be cl-supercontinuous. Then gof is continuous.*

Proof. It is immediate in view of the above lemma and Theorem 2.2. However, if $f : X \to Y$ is slightly continuous and $gof : X \to Z$ is continuous, the function $g : Y \to Z$ may not be cl-supercontinuous. □

Example 2.18. Let $X = \{a, b\}$ endowed with discrete topology.
Let $Y = \{c, d\}$, $\tau = \{\phi, Y, \{c\}\}$. Let $f : X \to Y$ be defined by $f(a) = c$, $f(b) = d$. Let $Z = \{e, f\}$, $\Im = \{\phi, Z, \{e\}\}$. Let $g : Y \to Z$ be defined by $g(c) = e$, $g(d) = f$. Then $f : X \to Y$ is slightly continuous and $g \circ f : X \to Z$ is continuous but $g : Y \to Z$ is not cl-supercontinuous.

3. cl-Quotient Topology and cl-Quotient Spaces

Let $f : X \to Y$ be a surjection from a topological space X onto a set Y. The quotient topology on Y is the largest topology on Y, which makes f continuous. Analogously, the largest topology on Y for which f satisfies a strong variant

of continuity yields a variant of quotient topology which in general is coarser than quotient topology. Such variants of quotient topology are dealt with in ([3] [4] [5] [9] and [13]) and interrelations among these are outlined in [7].
In the same spirit we define **cl-quotient topology** on Y as the finest topology on Y for which f is cl-supercontinuous. In this case the map f is called a **cl-quotient map**.

Theorem 3.1. *Let $f : X \to Y$ be a cl-quotient map. Then a function $g : Y \to Z$ is continuous if and only if $g \circ f$ is cl-supercontinuous.*

4. Topological Properties and cl-Supercontinuity

Theorem 4.1. *Let $f : X \to Y$ be a cl-supercontinuous open bijection. Then X and Y are homeomorphic zero-dimensional spaces.*

Proof. Let $x \in X$ and let U be an open set containing $f(x)$. Since f is an open map, $f(U)$ is an open set containing $f(x)$. In view of cl-supercontinuity of f, there exists a clopen set V containing x such that $f(V) \subset f(U)$. This implies $x \in f^{-1}(f(V)) \subset f^{-1}(f(U))$. Since f is a bijection, $f^{-1}(f(V)) = V$ and $f^{-1}(f(U)) = U$, so $x \in V \subset U$. Thus the space X has a base of clopen sets and so it is zero-dimensional. Since zero-dimensionality is a topological property and f is a homeomorphism, Y is also zero-dimensional. □

Definition 4.2. *A function $f : X \to Y$ is said to be a **cl-homeomorphism** if f is a bijection such that both f and f^{-1} are cl-supercontinuous.*

Theorem 4.3. *Let $f : X \to Y$ be a cl-homeomorphism from a space X onto a space Y. Then both X and Y are homeomorphic zero-dimensional spaces.*

Definition 4.4. *A topological space X is said to be **ultra-Hausdorff*** [15] *if each pair of distinct points are contained in disjoint clopen sets.*

Theorem 4.5. *Let $f : X \to Y$ be a cl-supercontinuous injection.If Y is a T_0-space, then X is ultra-Hausdorff.*

Proof. Let x_1 and x_2 be two distinct points in X. Then $f(x_1) \neq f(x_2)$. Since Y is T_0-space, there exists an open set V containing one of the points $f(x_1)$ or $f(x_2)$ but not the other. For definiteness assume that $f(x_1) \in V$. Since f cl-supercontinuous, in view of Theorem 2.2 $f^{-1}(V)$ is cl-open containing x_1 but not x_2. Hence, there exists a clopen set $U \subset f^{-1}(V)$ containing x_1 but not x_2. Then U and $X \setminus U$ are disjoint clopen sets containing x_1 and x_2 respectively. Hence X is ultra-Hausdorff. □

Definition 4.6. *A space X is called **mildly compact*** [15] *if every clopen cover of X has a finite subcover.*

In [14] Sostak calls mildly compact spaces as **clustered spaces**.

Theorem 4.7. *Let $f : X \to Y$ be a cl-supercontinuous function from a clustered space X onto Y. Then Y is compact. Further, if Y is Hausdorff, then f is a cl-closed function.*

Corollary 4.8. *Let $f : X \to Y$ be a cl-supercontinuous surjection from a connected space X onto Y. Then Y is a connected, compact space.*

The following result shows that there exists no nonconstant cl-supercontinuous function from a connected space into a T_0-space.

Theorem 4.9. *Let $f : X \to Y$ be a non constant cl-supercontinuous function. If Y is a T_0-space, then X is disconnected.*

5. Change of Topology

If in the domain of a cl-supercontinuous functions f, it is defined another topology in an appropriate way, then f is simply a continuous function. Let (X, τ) be a topological space, and let β denote the collection of all clopen subsets of (X, τ). Since the intersection of two clopen sets is a clopen set, the collection β is a base for a topology τ^* on X. Clearly $\tau^* \subset \tau$. The space (X, τ) is zero-dimensional if and only if $\tau^* = \tau$.
Throughout the section, the symbol τ^* will have the same meaning as in the above paragraph.

Theorem 5.1. *A function $f : (X, \tau) \to (Y, \Im)$ is cl-supercontinuous if and only if $f : (X, \tau^*) \to (Y, \Im)$ is continuous.*

Many of the results studied in preceding sections follow now from above theorem and the corresponding standard properties of continuous functions.

Theorem 5.2. *Let (X, τ) be a topological space. Then the following statements are equivalent.*

(a) *(X, τ) is zero-dimensional.*
(b) *Every continuous function from (X, τ) into a space $(Y, \Im)$ is cl-supercontinuous.*

Proof. $(a) \Rightarrow (b)$ is obvious.
$(b) \Rightarrow (a)$: Take $(Y, \Im) = (X, \tau)$. Then the identity function 1_x on X is continuous, and hence cl-supercontinuous. Hence by Theorem 5.1, $1_x : (X, \tau^*) \to (X, \tau)$ is continuous. Since $U \in \tau$ implies $1_x^{-1}(U) = U \in \tau^*$, $\tau \subset \tau^*$. Therefore $\tau^* = \tau$, and so (X, τ) is a zero-dimensional. □

Theorem 5.3. *Let $f : (X, \tau) \to (Y, \Im)$ be a function. Then*

(a) *f is slightly continuous if and only if $f : (X, \tau) \to (Y, \Im^*)$ is continuous.*
(b) *f is cl-open if and only if $f : (X, \tau^*) \to (Y, \Im)$ is open.*

In the light of Theorems 5.1 and 5.3 Theorem 2.10 can be restated as follows. If $f : (X, \tau^*) \to (Y, \Im)$ is a continuous open surjection and $g : (Y, \Im) \to (Z, \nu)$ is a function, then g is continuous if and only if $g \circ f$ is continuous and Theorem 2.17 is simply the result that the composition $g \circ f$ of the continuous functions $f : (X, \tau) \to (Y, \Im^*)$ and $g : (Y, \Im^*) \to (Z, \nu)$ is continuous.
Moreover, cl-quotient topology on Y determined by $f : (X, \tau) \to Y$ in Section 3 coincides with usual quotient topology on Y determined by $f : (X, \tau^*) \to Y$.
Finally we point out that in certain situations, in contrast to continuous functions, the set L of all cl-supercontinuous functions is closed in the topology of

pointwise convergence (see [6], [10]). For example, if X is sum connected [2] (e.g. connected or locally connected) and Y is Hausdorff, then the set $L(X,Y)$ of all cl-supercontinuous functions is closed in Y^X in the topology of pointwise convergence. In particular, if X is connected (or locally connected) and Y is Hausdorff, then the pointwise limit of a sequence of cl-supercontinuous functions is cl-supercontinuous.

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References

[1] R. C. Jain, *The role of regularly open sets in general topology*, Ph.D. thesis, Meerut Univ., Institute of Advanced Studies, Meerut, India (1980).

[2] J. K. Kohli, *A class of spaces containing all connected and all locally connected spaces*, Math. Nachricten **82** (1978), 121–129.

[3] J. K. Kohli and R. Kumar, *Z-supercontinuous functions*, Indian J. Pure Appl. Math. **33** (7) (2002), 1097–1108.

[4] J. K. Kohli and D. Singh, *D-supercontinuous functions*, Indian J. Pure Appl. Math. **32** (2) (2001), 227–235.

[5] J. K. Kohli and D. Singh, *D_δ-supercontinuous functions*, Indian J. Pure Appl. Math. **34** (7) (2003), 1089–1100.

[6] J. K. Kohli and D. Singh, *Function spaces and strong variants of continuity*, Applied Gen. Top. (accepted).

[7] J. K. Kohli, D. Singh and Rajesh Kumar, *Some properties of strongly θ-continuous functions*, Communicated.

[8] N. Levine, *Strongly continuity in topological spaces*, Amer. Math. Monthly **67** (1960), 269.

[9] B. M. Munshi and D.S. Bassan, *Super-continuous mappings*, Indian J. Pure Appl. Math. **13** (1982), 229–236.

[10] S. A. Naimpally, *On strongly continuous functions*, Amer. Math. Monthly **74** (1967), 166–168.

[11] T. Noiri, *Supercontinuity and some strong forms of continuity*, Indian J. Pure. Appl. Math. **15** (3) (1984), 241–250.

[12] I. L. Reilly and M. K. Vamanamurthy, *On super-continuous mappings*, Indian J. Pure. Appl. Math. **14** (6) (1983), 767–772.

[13] D. Singh, *D^*-supercontinuous functions*, Bull. Cal. Math. Soc. **94** (2) (2002), 67–76.

[14] A. Sostak, *On a class of topological spaces containing all bicompact and connected spaces*, General Topology and its Relation to Modern Analysis and Algebra IV: Proceedings of the 4th Prague Topological Symposium, (1976), Part B, 445–451.

[15] R. Staum, *The algebra of bounded continuous functions into a nonarchimedean field*, Pac. J. Math. **50** (1) (1974), 169–185.

[16] L. A. Steen and J. A. Seeback, Jr., *Counter Examples in Topology*, Springer Verlag, New York, 1978.

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