Experimental realization of broadband tunable resonators based on anisotropic metafluids

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This letter demonstrates the mechanical tuning of modes confined in two-dimensional acoustic cavities based on anisotropic metafluids. The employed metafluids have been designed with effective parameters such that the radial component of the sound speed tensor keeps constant and the acoustic impedance remains finite along the tuning. It is shown that mode frequencies can be mechanically down shifted to extremely low values. Experiments confirm the model predictions and let us to conclude that this type of resonators can be employed for the miniaturization of standard resonators based on isotropic fluids such as air. © *2011 American Institute of Physics*. [doi:10.1063/1.3599849]

Acoustic metamaterials or metafluids are artificial structures whose dynamical behavior is new in comparison with that exhibited by materials available in nature. Metafluids are made of subwavelength solid units and their acoustic parameters can be obtained by effective medium theories. Those with negative parameters (negative bulk modulus, negative mass density, or both simultaneously) have been already demonstrated,[1–3] and many interesting applications have been envisaged based on them. However, we focus on metafluids having anisotropic dynamical mass density, a property required to build fascinating devices such as cloaking shells,[4] acoustic hyperlenses,[5] or radial sonic crystals.[6,7]

Anisotropic dynamical mass density in two-dimensions (2D) has been obtained by using nonsymmetric lattices (i.e., other than hexagonal or square) of rigid inclusions in air.[8–10] Previously, Bradley[11] demonstrated that multilayered fluid–fluid structures with different dynamical mass densities can be obtained by using planar waveguides with corrugated surfaces. These one-dimensional (1D) periodic systems behave in the low frequency region such as an effective fluid with a tensorial mass density with diagonal components ρ_{xx} and ρ_{yy}, where the x-direction defines the 1D periodicity. This approach has been applied to produce cylindrical mass anisotropy by using radially corrugated surfaces,[12] being ρ_r and ρ_θ the components of the mass density tensor. Dynamical mass anisotropy along the Cartesian directions has been also recently demonstrated.[13]

This letter shows that anisotropic metamaterials can be used to build broadband acoustic resonators where modes with extremely low frequencies can be attainable. This proposal has been verified by constructing a 2D acoustic cavity consisting of a specially designed substrate composed of a corrugated bottom surface and a flat top plate, both made of aluminum (Al) and separated by a spacer filled with air. It is shown that the confined modes can be mechanically adjusted within a broad range of frequencies. It is also observed that the frequency of the dipolar mode reaches a very low value, which is mainly determined by the precision of our drilling mechanical tools.

Cylindrically anisotropic metamaterial periodically corrugated along the radius substrate and a flat top plate has been reported in Ref. 12. Figure 1(a) depicts a schematic view of the substrate and a picture of an actual sample, which consists of circular grooves of certain thickness, d_1, separated by a distance, d_2, with periodicity, $d=d_1+d_2$. The resulting waveguide (substrate, top, and air) behaves as an anisotropic metafluid characterized by a tensorial dynamical mass density with components ρ_r and ρ_θ and a scalar bulk modulus B.[8] If a harmonic sound wave with linear frequency ν propagates through this effective fluidlike material, the pressure field is described by a linear combination of real-order Bessel functions[14]

$$P(r,\theta;\nu)=\sum_{q=0}^{+\infty} A_q J_{q\gamma}(2\pi\nu r/c_r)\ \cos(q\theta+\phi_q), \qquad (1)$$

where q are integers, $\gamma=\sqrt{\rho_r/\rho_\theta}$ is the anisotropy factor, and $c_r=\sqrt{B/\rho_r}$ is the radial component of the sound speed tensor. Finally, A_q and ϕ_q are indefinite constants.

Acoustic cavities can be easily made from these structures by enclosing the waveguide inside a circular region of radius R_0. The frequencies of the resonant modes in these 2D acoustical cavities are obtained from the rigid wall condition

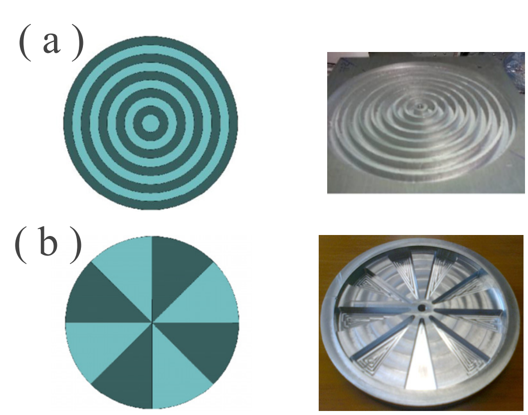

FIG. 1. (Color online) (a) Schematic view of a radially periodic structure employed in Ref. 12 to develop cylindrically anisotropic metafluids (left) and its physical realization (right). (b) Schematic view of the periodic structure studied in this letter (left) together with its physical realization (right).

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at the cavity boundary,[15] i.e., $|\partial P/\partial r|_{r=R_0}=0$. The following transcendental equation was obtained:[12]

$$J'_{q\gamma}(2\pi\nu_{nq}R_0/c_r)=0, \quad n=1,2,\ldots, \tag{2}$$

where $J'_{\gamma q}(\cdot)$ stands for the derivative of Bessel function of order $q\gamma$.

Since we are interested in low frequency modes, we use the small-argument approximation for the Bessel functions[16] to obtain the lowest zero of Eq. (2),

$$\nu_q \simeq \frac{c_r}{\pi R_0}\left(\frac{\gamma q}{2}\right)^{1/2} q=0,1,2,\ldots. \tag{3}$$

Note that subindex n has been suppressed since, in this letter, we will only analyze q-modes of lower order, i.e., $n=1$ for every q. This equation shows that, for fixed R_0 and q, there are two options to obtain low-frequency resonances; one is to reduce the speed component c_r and the other is to decrease the anisotropy factor γ. Both ways are possible within the approach presented here, but decreasing c_r hinders the propagation of sound as well as increases the cylinder's surface impedance, which is a fundamental drawback for the possible applications of these structures as acoustic metamaterials.[17] Therefore, to obtain modes with extremely low frequencies we consider that reducing γ in Eq. (3) is the best option.

Recall that structures shown in Fig. 1(a) have been modeled as radially periodic fluid–fluid systems, which for the simplified case of layers with equal thickness, $d_1=d_2=d/2$, the sound speed components and the anisotropy factor are[12]

$$c_r^* = c_b\frac{2}{\left[1+\dfrac{h_1}{h_2}\right]^{1/2}\left[1+\dfrac{h_2}{h_1}\right]^{1/2}}; \quad c_\theta^*=c_b; \quad \gamma^*=c_\theta^*/c_r^*, \tag{4}$$

where c_b is the airborne sound speed, h_1 is the separation between substrate and top plate, and h_2 is the distance between the bottom of the grooves drilled in the substrate and the top plate. These parameters were obtained under the assumption that Al is perfectly rigid and, therefore, sound waves only propagate through air.

Starting from these structures it is possible to obtain the type of structures shown in Fig. 1(b) by applying a transformation that switches the $\hat{r}$ and $\hat{\theta}$ axes. The components of the transformed mass density tensor are obtained by changing $\rho_\theta^*\to\rho_r$ and $\rho_r^*\to\rho_\theta$. Moreover, the bulk modulus only depends on the filling fraction, which is the same in both structures and then $B^*\to B$. Therefore, the sound speed tensor and the anisotropy factor are now,

$$c_r=\sqrt{\frac{B^*}{\rho_\theta^*}}=c_b; \quad c_\theta=\sqrt{\frac{B^*}{\rho_r^*}}$$

$$=c_b\frac{2}{\left[1+\dfrac{h_1}{h_2}\right]^{1/2}\left[1+\dfrac{h_2}{h_1}\right]^{1/2}}; \quad \gamma=\frac{c_\theta}{c_r}=\frac{1}{\gamma^*}. \tag{5}$$

These parameters have the property needed to safely obtain low frequency modes by decreasing the ratio h_1/h_2, that is to say, their sound speed component ρ_r is kept constant and the acoustic impedance $\rho_r c_r$ remains finite along the tuning. Thus, for structures such that $h_1\ll h_2$, $\gamma\approx 2\sqrt{h_1/h_2}$, and, therefore, a reduction in the ratio h_1/h_2 produces smaller values of γ. Since h_1/h_2 is determined by the distance between substrate and top plates, a redshift of resonant frequencies should be produced by approaching both plates toward each other.

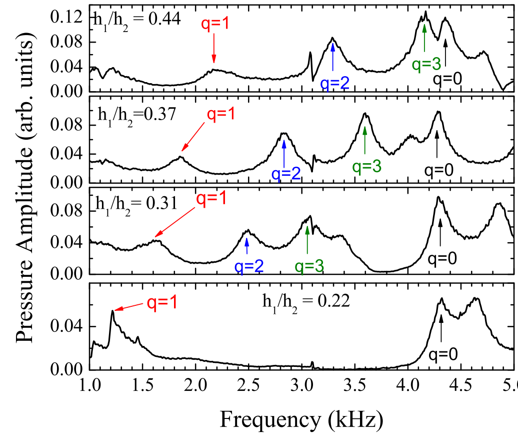

FIG. 2. (Color online) Spectra taken inside the cavity made of corrugated samples with different ratios h_1/h_2. The peaks correspond to resonances with different polar symmetry. Note that the frequency of dipolar mode ($q=1$) is redshifted by decreasing the ratio h_1/h_2 while the frequency of monopolar mode ($q=0$) is kept constant.

To demonstrate the previous predictions a sample cavity of radius $R_0=52$ mm was constructed with an adjustable top. The bottom surface, which is shown in Fig. 1(b), consists of nine 5 mm deep triangular shaped ditches drilled in the Al plate. The distance h_1 was adjustable by a screw placed at the cavity center. Acoustic characterization has been made as described in Ref. 12. In brief, an external loudspeaker is used to excite white noise inside the cavity, and the cavity response is measured by two microphones, one located at the cylinder axis and another off-axis to check the mode's symmetry. The air sound speed, c_b, was first measured in an empty cavity (no metamaterial inside) with the same diameter and the value 344.46 m/s was obtained.

Figure 2 shows the spectra measured for several cavity's ratios h_1/h_2. It is observed that the monopolar resonance keeps its position when the ratio decreases while higher order resonances ($q=1,2,3$) are redshifted. This behavior is physically understood since the frequency of the $q=0$ mode only depends on c_r, which according to Eq. (5) is equal to c_b. On the other hand, higher order modes are shifted down in frequency as predicted by Eq. (3) when $h_1/h_2\to 0$ and, therefore, resonances with near-zero frequency should be theoretically possible.

In the limiting case in which $h_1\approx 0$ (last spectrum in Fig. 2) all the nonmonopolar frequencies should converge to that of the corresponding $q=0$ mode since $J'_{q\gamma}(x)\to J'_0(x)$ when $\gamma\to 0$. In other words, a zero-frequency resonance should exist, in principle, for $q=0$ since $J'_0(0)=0$. But this resonance will be never observed due to the fact that the ideal case $h_1=0$ (perfect contact between substrate and top plates) corresponds to nine uncoupled triangular cavities. However, this limit is impossible to achieve by any sample because of unavoidable defects or inhomogeneities existing on both plate's surfaces produced during the fabrication process. For example, if we consider that the surface roughness due to the precision of the drilling machine is about ϵ, the minimum distance between both surfaces $h_1\approx 2\epsilon$, and the minimum

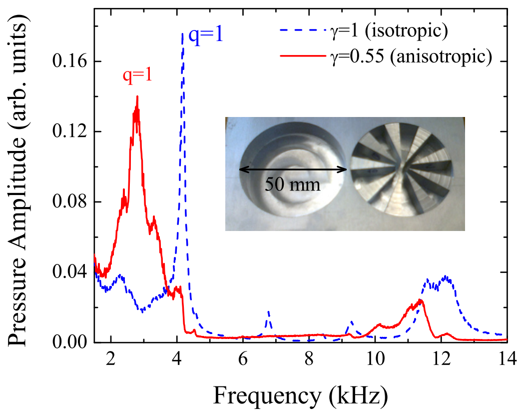

FIG. 3. (Color online) Spectra simultaneously taken in two acoustic cavities with identical diameters; one contains an isotropic fluid (air) and the other contains a metafluid with anisotropy factor $\gamma=0.55$. The inset shows a picture of the cavities' substrates both drilled on the same aluminum plate.

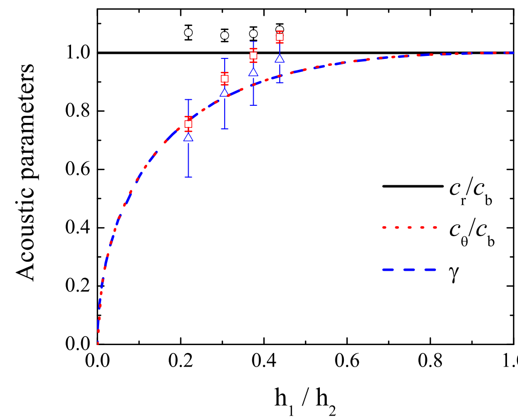

FIG. 4. (Color online) Symbols with error bars represent experimental data for the components of the sound speed tensor relative to that of the background (circles and squares) and the anisotropy factor (triangles). Lines give the values obtained by the analytical model. Note that lines describing the behavior for γ and c_θ/c_b overlap.

resonant frequency that would be measured corresponding to the $q=1$ mode is

$$\nu_1 \approx \frac{c_b}{\pi R_0}\left(\frac{2\epsilon}{h_2}\right)^{1/4}. \tag{6}$$

For the dimension of our cavity sample and considering the maximum precision of the drilling tool $\epsilon \approx 0.01$ mm, the lowest attainable frequency is $\nu_1 \approx 0.5$ kHz.

For a further support of the effect discussed above, we have fabricated in the same plate two acoustic cavities with equal radii ($R_0=25$ mm) but containing two different fluid-like materials. The substrates of both cavities are shown in the inset of Fig. 3. The cavity on the left is a hollow cavity that contains air (an isotropic fluid). The one on the right contains an effective anisotropic fluid made of an artificial structure with parameters described by Eq. (5).

Spectra for both cavities were simultaneously taken and are depicted in Fig. 3, where it is noticed the redshift of the dipolar mode when it is confined in the anisotropic cavity. From the natural frequency shift and the anisotropy factor we can calculate an "apparent diameter," D', which is defined as the diameter of a hollow cavity having the dipole confined at the same frequency than that of the anisotropic cavity. It is given by the condition $J'_\gamma(2\pi\nu_{nq}R_0/c_r)=0$, which for the anisotropic cavity under study gives the value $D'=75$ mm. This value, which is 1.5 times larger than its actual diameter, let us to conclude that acoustic cavities made of anisotropic fluidlike materials allow the confinement of low frequency modes in reduced dimensions, smaller than that confined in cavities based on isotropic materials.

To verify the model employed in the description of the metafluids studied here, we have extracted the parameters from the measured frequencies by using the expressions in Eq. (2). The results are given by symbols in Fig. 4 and are compared with that (continuous lines) obtained from Eq. (5). Note that, discrepancies between model and experimental values for γ and c_θ are larger for larger values of ratio h_1/h_2, which might be due to the fact that Eq. (2) is more accurate for lower frequency values. Also, the model gives a systematic underestimation of about 6% in the value of c_r, which might be due to some artifact in the experimental setup, such as the existence of a rigid cylinder at the cavity center (the screw). Moreover, another source of error in the model is the fact that it does not consider any interaction between evanescent modes generated in the structure.

In summary, acoustic metamaterials or metafluids with cylindrical mass anisotropy have been employed to build mechanically tunable 2D acoustic resonators. Anisotropic metamaterials are designed and constructed to theoretically allow the tuning of the resonant frequencies up to near-zero values. The predicted near-zero-frequency limit could not be observed due to sample imperfections during fabrication process. Nevertheless, it has been demonstrated that resonators based on anisotropic metafluids have their modes localized at lower frequencies than that based on isotropic materials with the same dimensions. We conclude that anisotropic metafluids such as those analyzed here can be employed in the miniaturization of standard acoustic resonators consisting of isotropic fluids such as air.

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