Enhanced transmission band in periodic media with loss modulation

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We study the propagation of waves in a periodic array of absorbing layers. We report an anomalous increase of wave transmission through the structure related to a decrease of the absorption around the Bragg frequencies. The effect is first discussed in terms of a generic coupled wave model extended to include losses, and its predictions can be applied to different types of waves propagating in media with periodic modulation of the losses at the wavelength scale. The particular case of sound waves in an array of porous layers embedded in air is considered. An experiment designed to test the predictions demonstrates the existence of the enhanced transmission band.

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Wave propagation in periodic media has become a subject of intensive study with numerous applications in different fields. The simplest form of periodic media consist of alternating material layers with different properties (such as the refraction index in optics, or the density or elasticity parameters in acoustics) forming a layered medium, also referred as 1D crystal or superlattice. Originally formulated to explain the propagation of electrons in solids[1], the basic theory of wave propagation in layered media was soon extended to optics[2] and acoustics[3,4]. Most of the previous work on periodic media focused on conservative systems where waves can be reflected (at the bandgaps), deflected, scattered, or even localized inside the crystal. However, waves cannot be absorbed unless dissipation is considered in the system. While dissipation is an inherent property of all forms of matter, few attention has been paid to its effects in periodic media. Moreover, especially in real experiments, often one or more of the constituent materials present some non-negligible losses in the frequency range of interest.

Light and sound waves behave in the same manner in linear media, obeying similar wave equations. This has inspired a number of analogies between both fields. However, the motivation for the study of losses in acoustics and optics may be different. In optics, where efforts are devoted to minimize losses, dissipation in periodic systems has been considered recently[5–10]. While in Refs. [5] and [6] absorption is reduced in a multilayered magneto-photonic crystal, in Refs. [7] and [8] enhanced transmission through a stack of dielectric layers having contrast only in attenuation is reported. Extensions to two-dimensional (2D) modulation of losses has shown to provide nontrivial light beam propagation effects, analogous to flat photonic crystal lensing reported in conservative systems[9,10]. In acoustics the situation is different, since achieving maximum absorption is often the goal. The effect of viscoelastic losses on phononic crystals was first discussed in Ref. [11], and more recently in Refs. [12–14], in terms of the modification of dispersion relations. Damping of elastic waves in solids phononic crystals has also been discussed in [15] and [16]. In the audible regime, viscothermal losses dominate, and absorption is mainly achieved by using resonators or porous materials[17]. The behaviour of lossy periodic media for waves near Bragg resonances is much less known than the long-wavelength limit. In this regime, there are studies about wave propagation in acoustic absorbing media with rigid periodic inclusions[18], and in 2D arrays made of absorbent[19] and, absorbent and resonant scatterers embedded in air[20]. The combination of periodicity and absorption in substructured materials produces complete absorption of sound with a broadband response and functional for any direction of incident radiation[21].

In this work we investigate the wave propagation within a layered material with periodically distributed losses. We show how the periodicity of the absorbing media can modify the global absorption of the system as well as its reflection and transmission properties. The main prediction is a simultaneous increase of transmission and reflection around the Bragg frequency, an anomalous behavior in contrast to classical, conservative bandgaps that always result in a decrease of transmission. First, a generic model based on the coupled-mode theory and valid for different types of waves (light, sound or matter waves) and media is presented, and its transmission/reflection characteristics are analytically deter-

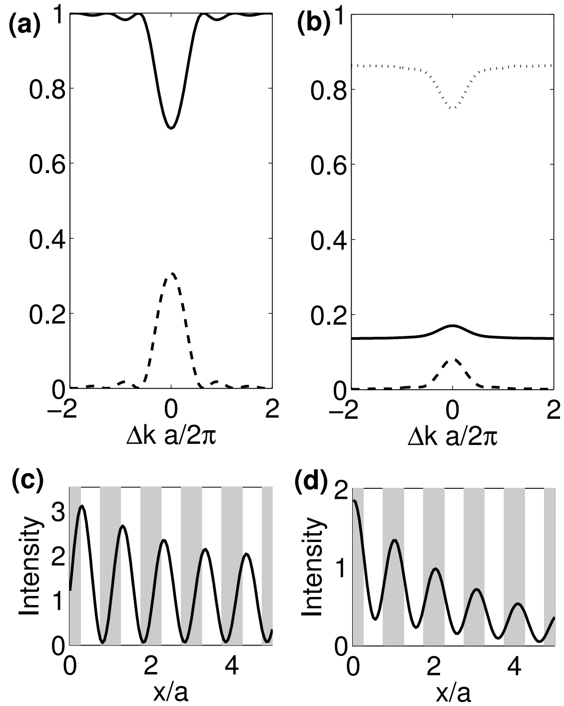

FIG. 1. Transmission (solid lines) reflection (black dotted lines) and absorption (gray dotted lines) spectra for waves in a periodic structure (5 periods, $L/a = 5$) as calculated from Eqs. (3) and (4) for (a) conservative system (with coupling $ma = 0.125$ and no losses $\gamma a = 0$) showing the well known band-gaps, (b) periodic system (with pure imaginary coupling valued $ma = i0.125$ and losses $\gamma a = -0.2$) predicting the anomalous transmission. (c) and (d) show the total intensity at the Bragg frequency, $\Delta k = 0$, for the configurations shown in (a) and (b) respectively. Grey areas represent the absorbing material in (d).

mined. Next, we particularize the study to the case of sound waves propagating in a 1D periodic structure of porous layers embedded in air, which is theoretically and experimentally examined. The anomalous of transmission band around bandgap frequencies is experimentally observed, showing good agreement with theory even for a minimal number of layers.

Waves in layered media can be studied by using different theoretical tools. One approach very popular in photonics is the coupled-mode theory[22]. Here we extend the theory to include the effect of losses, and calculate its influence in the transmission/reflection spectrum. Consider a medium formed by a finite number of lossy parallel identical and equidistant layers irradiated by an incident plane wave. The total field is composed of forward and backward propagating waves $P = A(x)e^{ik_Bx-i\omega t} + B(x)e^{-ik_Bx-i\omega t} + c.c.$, which amplitudes are normalized so that their absolute square is proportional to the energy flux in the corresponding direction. $k_B = \pi/a$ is the Bragg wavenumber (the edge of Brillouin zone, being a the lattice constant of the system) and ω is the frequency. Forward and backward waves are coupled by the modulation. If the contrast of impedances between layers is small, and for frequencies near a Bragg resonance, the dynamics of the forward and backward waves can be approximately described by the dissipative coupled-mode equations,

$$\frac{dA}{dx} = i\Delta kA + mB + \gamma A,$$
$$-\frac{dB}{dx} = i\Delta kB + mA + \gamma B, \qquad (1)$$

where $\Delta k = k - k_B$ is the detuning from the Bragg wavenumber, m is the coupling between forward and backward waves which is generally complex: real for reflections from conservative (rigid or penetrable without losses) materials, and imaginary for reflections from purely absorptive media. A complex value of m allows representing any realistic material. The coupling coefficient m is related to the impedance mismatch between the absorber and the host medium. If the reflection coefficient from medium 1 to medium 2 is r_{12} and $r_{21} = -r_{12}$, and considering the same acoustic thickness (or equivalently, the optical path) d for both materials, the coupling coefficient is: $m = (r_{12} - r_{21})/d = 2r_{12}/d$. For the case of an acoustic wave: $r_{12} = (Z_2 - Z_1)/(Z_2 + Z_1)$, where Z_i stands for the impedance of the i-th medium. Finally γ is the gain coefficient, being negative for the case of a lossy media. Its worth noting that γ is always negative for an acoustic media ($\gamma < 0$, since there are no gain acoustic materials). Note that the following relation holds $|\gamma| > |Im(m)|$.

The solutions of Eqs. (1) are exponentially growing/decaying oscillating waves, $A(x), B(x) = e^{\lambda x}$, where λ are the complex eigenvalues of the matrix of the coefficients of Eqs. (1), which read

$$\lambda_\pm = \pm\sqrt{(\gamma + i\Delta k)^2 + m^2}. \qquad (2)$$

For a finite system of length L, formed by N layers, transmission and reflection coefficients can be obtained analytically by imposing boundary conditions at the entrance face ($x = 0$) for the forward field, $A(x = 0) = 0$, and at the rear face ($x = L$) for the backward field $B(x = L) = 0$. This leads to

$$T = \frac{\lambda}{\lambda\cosh(\lambda L) - (\gamma + i\Delta k)\sinh(\lambda L)} \qquad (3)$$

$$R = \frac{m\sinh(\lambda L)}{\lambda\cosh(\lambda L) - (\gamma + i\Delta k)\sinh(\lambda L)} \qquad (4)$$

with λ given by Eq. (2) with the negative sign (physical solutions of the problem).

These expressions can be used to evaluate the response of the structure in two opposite cases: the well-known conservative periodic system $\gamma = 0$ and pure real modulations parameter and a fictional material called here purely absorptive material, that is a medium with the same real part of the impedance as the host, but a non-null imaginary part, i.e. pure imaginary m and negative γ. The latter case is analogous to that considered for photonics in Refs. [7] and [8]. As it is well known, for conservative periodic materials, the waves around the Bragg frequency $f_B = c/2a$ (being c the velocity of the wave in the medium) are efficiently back reflected due to Bragg resonance and transmission is correspondingly reduced, as shown in Fig. 1(a).

However, in the case of lossy periodic media, the situation is different since the material parameters may have a complex value due to dissipation. In the ideal case of a purely absorbent material, we observe that an anomalous transmission is maximum at Bragg resonance ($\Delta k = 0$), as observed in Fig. 1(b). The origin of such anomalous phenomenon is explained in Figs. 1(c) and 1(d), where the field distribution along the structure is shown for both cases. For a purely absorbent structured material, at these frequencies, the total field within the structure partially forms a standing wave, with the nodes of the particle velocity (maximum values of the field) located precisely inside the absorbing media. As the nodes correspond to low particle velocity, there is few energy to be absorbed. As a consequence, such a configuration results in smaller absorption: both forward as well as backward waves are less absorbed, and the overall transmission is increased as well as the absorption is reduced as shown in Fig. 1(b).

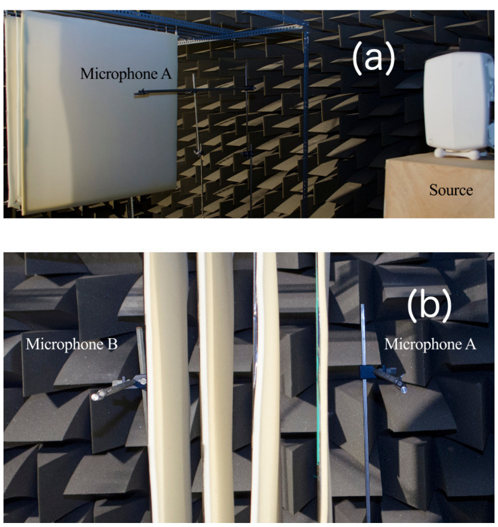

FIG. 2. (Color online) (a) Experimental set-up, consisting in an array of four plates of porous material; showing the source, a loudspeaker located in front of the structure, and the microphone to measure intensity at either side of the structure. (b) View of the system from a different angle.

The coupled wave formulation presented above is independent of the particular type of wave. Then, the coefficients are generic and do not contain information on the physical characteristics of the considered system. We concentrate now in the particular case of sound waves propagating through periodically spaced porous layers, of thickness D embedded in a fluid media (air) being a the distance between the center of two consecutive layers (lattice constant). This study will be used to check the predictions of the general model as well as to compare with experiments.

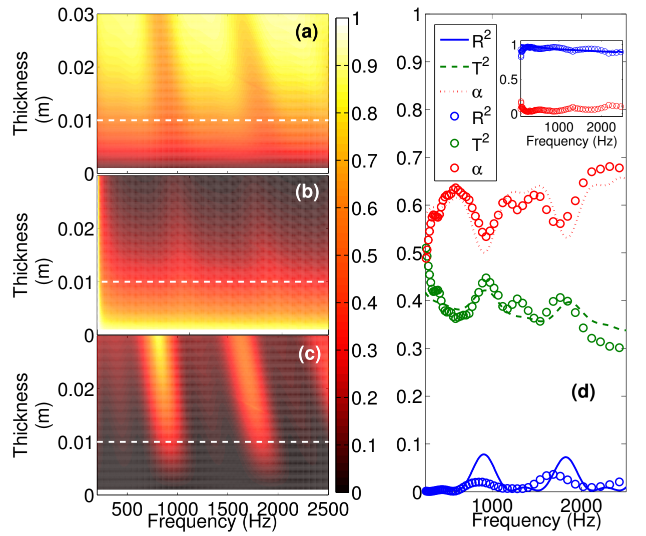

FIG. 3. (Color online) (a), (b) and (c) Dependence of the absorption, reflection and transmission coefficients on the thickness of the porous layer and on the frequency for the stratified media, calculated with TMM for $N = 3$ layers. (b) Reflection (blue continuous), Transmission (green dashed) and Absorption (red dotted) of our system (corresponding to the white dashed line in (a)-(c)). Continuous lines represent the theoretical predictions and circles represent the experimental results. Inset shows the reflection (blue continuous) and absorption (red dotted) coefficients of a single porous layer for its characterization using the ISO-10534-2.

An experiment was designed to check the predictions of anomalous transmission around Bragg frequencies. The set-up consists of a set of 3 to 5 parallel porous layers of $D = 8$ mm thickness embedded in air, as shown in Fig. (2). The lattice constant was chosen as $a = 20$ cm. A loudspeaker was placed in front of the first layer in such a way that plane waves propagate through the system. All the measurements were conducted in an anechoic chamber in order to avoid unwanted reflections. The coefficients (reflection, transmission and absorption) were calculated from the acoustic pressure measurements registered by two microphones, in both sides of the periodic structure. The spectral characteristics were measured using the above described experimental scheme. Experimentally, we determined the intensity coefficients by measuring the sound field before (reflection R) and after (transmission T) the structure. Finally, by energy balance, the absorption coefficient is obtained as $\alpha = 1 - |R|^2 - |T|^2$.

We consider here the most general case in which the frame of the porous material presents an elastic behaviour, so Biot's theory can be used to characterize the porous material. The layered material used in experiments is analytically characterized by the transfer matrix method (TMM) described in Ref. [17]. We consider that the layered structure is laterally infinite (1D) and made of homogeneous and isotropic porous layers embedded in air. We calculate the transfer matrices in the porous medium where two compressional waves and one shear wave can be supported and in the fluid medium with only one compressional wave. All these waves are coupled by the boundary conditions and the result is a global transfer matrix which gives the propagation properties of the stratified media made of N layers, and in particular its reflection and transmission coefficients.

Densitiy (kg/m^3), ρ	50
Porosity, ϕ	0.97
Young's Modulus (kPa), E	150
Poisson's coefficient	0.35
Tortuosity, α_∞	1
Flow resistivity, σ	13000
Characterisitic length (m), Λ	120x10^{-6}
Characterisitic thermal length (m), Λ'	200x10^{-6}

TABLE I. Physical parameters of the porous material used in the experiments and numerics.

In a first step, the material has been characterized. Parameters of the material are shown in Tab. I. These parameters have been used to evaluate the transmission and absorption coefficients of the porous layer using the TMM. These properties are shown in the inset of Fig. 3(d), showing that the parameters of Tab. I represents in good agreement the transmission and absorption properties obtained using the standard ISO-10534-2. We can see that the absorption of the porous material is very low, therefore the effective impedance of the porous layer is similar to that of the air. This situation is optimal to allow transmission with small but enough losses to induce the anomalous properties of a layered media made of layers of this porous material.

Once the material is characterized, we use the TMM to evaluate the properties of a layered material made of 3 porous layers embedded in air. The dependence of the absorption, α, reflection, $|R|^2$, and transmission, $|T|^2$, coefficients on the thickness of the porous layer, d, and on the frequency, f, are shown in Figs. 3(a), 3(b) and 3(c) respectively. We can observe, as predicted previously by the general coupled-mode model, the usual increase of the reflection in the band-gap and the anomalous increase (decrease) of the transmission (absorption) at frequencies around the band gap ($f_B = 850$ Hz ($f_B = 1700$Hz) for the first (second) band gap).

Finally, we particularize for the case we have in the experimental set-up. Figure 3(d) shows the comparison between the numerical predictions, obtained by applying the TMM and the experimental results. As predicted, maxima of transmission and reflection are observed at Bragg frequencies and, as a consequence, at these frequencies the structure is absorbing less energy.

We determine transmission and reflection of waves in a general layered lossy structure and measure it experimentally in a particular acoustic system. The study indicates the existence of spectral regions of enhanced and reduced overall absorption with anomalous transmission around the band gap. A simple couple mode theory is proposed to explain these results, which is essentially a forward wave linearly coupled with the backward wave. Depending on the character of the systems (rigid, lossy, or complex), the coupling coefficient is set (real, imaginary or complex), which also captures the above predicted and measured spectral characteristics. In good agreement with the TMM predictions, we experimentally observe that the transmission of sound waves trough a periodic arrangement of absorbing plates is enhanced at resonance. Such anti-bandgap effect is expected to be generic for any kind of waves in a periodic modulation of losses on the wavelength scale, at the Bragg frequency.

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