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Boulvert, J.; Humbert, T.; Romero-García, V.; Gabard, G.; Fotsing, ER.; Ross, A.; Mardjono, J.... (2022). Perfect, broadband, and sub-wavelength absorption with asymmetric absorbers: Realization for duct acoustics with 3D printed porous resonators. *Journal of Sound and Vibration*. 523:1-21. <https://doi.org/10.1016/j.jsv.2021.116687>



The final publication is available at

<https://doi.org/10.1016/j.jsv.2021.116687>

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Additional Information

Perfect, broadband and sub-wavelength absorption with asymmetric absorbers: realization for duct acoustics with 3D printed porous resonators

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Abstract

This article explores, theoretically and experimentally, asymmetric absorbers made of detuned and folded quarter wavelength resonators filled with air or porous materials. When used as acoustic lining in ducts, **their thickness can be sub-wavelength and they can be designed for perfect absorption in broad target frequency bandwidth**. The considered filling porous material can be easily 3D printed and is formed of a structured micro-lattice with variable lattice constant, allowing precise control of its acoustic properties. The underlying physics of asymmetric absorbers is discussed through a simplified analysis by means of the transfer matrix method. The behavior of the porous absorbers is also predicted by a mode-matching technique accounting for the possible couplings between the resonators. An absorber made of folded quarter-wavelength resonators is optimized, 3D printed and experimentally tested. The experimental results are in good agreement with the theory and show a mean absorption coefficient of 99% over **almost an octave and** below the quarter-wavelength frequency corresponding to the height of the absorber.

Keywords: duct acoustics, perfect absorption, asymmetric absorber, sub-wavelength, additive manufacturing, porous material

1. Introduction

A large variety of engineering applications involve duct noise problems such as car exhaust systems, mechanical housing ventilation systems, or aircraft engines. The width of the duct varies from less than a centimeter to meters. The noise frequency range depends on the main acoustic source, e.g., a car engine operating at a certain rotation speed, and of secondary sources such as the interaction of an air flux with the duct walls. Acoustic treatments can reduce noise emissions while allowing free air flow: mufflers are designed to reduce the transmission of sound waves, [1], anechoic terminations to avoid backward reflection at the end of a duct [2], and absorbers to simultaneously reduce the transmission while avoiding reflection, i.e., to absorb sound waves [3]. In this work, we focus on absorbers **targeting perfect** absorption, i.e., no reflection and no transmission, and more specifically on asymmetric absorbers, whose losses are carefully controlled by porous materials, that are flush mounted to the walls of a straight duct whose width is of the order of centimeters.

Homogeneous open-cell porous materials are widely used as acoustic treatments. Their behavior is well described by propagation models and their efficiency to operate as broadband acoustic absorbing panels has long been theoretically, numerically and experimentally demonstrated. They are sometimes used to control noise propagation through ducts, e.g., in porous car mufflers [1]. Nevertheless, when perfect absorption is targeted in ducted propagation, porous materials are often set aside so far due to physical reasons, mostly in favor of Helmholtz resonators (HRs) [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Using conventional porous materials in an optimization problem is frequently not fully satisfactory because of the lack of flexibility in the choice of their properties and their low efficiency in the sub-wavelength regime. Yet, recent advances in additive manufacturing now makes it possible to easily design and

20 manufacture porous materials having controlled pore shapes and dimensions, which allows to fully control their in-
21 trinsic acoustic behavior [14, 15, 16, 17, 18]. This way, the optimization of 3D printed acoustic treatments comprising
22 porous materials is not limited to a catalog of existing materials.

23 **Targeting perfect** absorption in a duct or waveguide requires either an asymmetric absorber [3, 10] or the use
24 of degenerate resonators with symmetric (monopolar) and antisymmetric (dipolar) resonances [8, 9, 10]. **So far,**
25 **asymmetric absorbers achieving very high absorption over a broad target frequency bandwidth are either compact but**
26 **make use of a very narrow waveguide (≤ 1 mm high) [5] or make use of a waveguide of the order of centimeters wide**
27 **but are not compact, i.e., the resonators forming the absorber are spaced by several centimeters [11].** In this work,
28 we explore the absorption capabilities of porous asymmetric absorbers because 3D printing allows to easily control
29 porous materials intrinsic properties. The studied porous absorbers are composed of locally reacting, detuned and
30 possibly folded quarter-wavelength resonators (QWRs) filled with porous materials with controlled pore size or air.
31 The absorbers are flush mounted to the walls of a straight waveguide of a few centimeters wide and of rectangular
32 cross-section.

33 **The objective of this work is twofold: (1) to show that compact absorbers can exhibit a very high absorption over**
34 **broad target frequency bandwidths when flush mounted on waveguides of the order of centimeters wide and (2) to**
35 **show that porous absorbers used in ducted problems can be efficient at frequencies below the quarter-wavelength**
36 **frequency corresponding to the height of the absorbers.**

37 In a first stage, the physical principles and limitations of asymmetric absorbers are described through a transfer
38 matrix method (TMM) analysis. This analysis leads to simple analytical conditions on the surface impedance to
39 obtain perfect absorption. This analysis also provides an expression for the minimal absorber length (dimension in
40 the direction of propagation of the incident wave) combining the frequency and the size of the duct. This expression
41 highlights that the length of the absorber can easily be lower than a quarter of the targeted wavelength λ in air and that
42 the real challenge is to consider a waveguide with a cross-section greatly larger than the area of the resonators surface
43 connected to the waveguide. In a second stage, the absorbers are optimized by making use of a model that considers
44 the potential coupling between the resonators for physically realistic waveguide configurations, demonstrating that
45 **an absorption coefficient of mean value close to unity over a broad target frequency bandwidth** can be achieved in a
46 duct using porous resonators **in their sub-wavelength regime** as acoustic lining. More specifically, a porous absorber
47 with a height of less than $\lambda/4$, is designed by using folded QWRs to lower the resonance frequencies of the resonators
48 without modifying their height. This absorber exhibits experimentally a 99% absorption **over almost one octave.**

49 The article is organized as follows: first, the porous asymmetric absorbers are introduced in Sec. 2. The considered
50 porous medium, its manufacturing and its acoustic behavior are presented, along with the geometry of the porous
51 absorbers including that of folded porous resonators. Then, the analytical and numerical modeling procedures of
52 waveguides lined by porous absorbers are described in Sec. 3. In Sec. 4, the analysis of the asymmetric absorbers
53 is performed using TMM predictions. The conditions to reach perfect absorption by means of asymmetric sound
54 absorbers are derived and expressed in terms of the resonators surface impedance. In Sec. 5, porous absorbers are
55 numerically optimized for realistic conditions. In addition, an example of a sub-wavelength absorber optimized for
56 perfect absorption **over a broad target frequency range** is manufactured and tested experimentally, showing a good
57 correlation between measured and predicted absorption.

58 2. Porous asymmetric absorbers

59 **This section introduces the asymmetric porous absorbers studied in this work and their constitutive elements:**
60 **QWRs filled with a porous medium or by air.**

61 2.1. Asymmetric absorber lining a duct of constant cross section

62 Perfect absorption in a duct propagation problem can be obtained when the propagation symmetry is broken [3],
63 when degenerated resonators are used [9], or when symmetry inversion frequency is tuned [19]. The simplest system
64 is thus an asymmetric absorber [3] that is composed of at least two different resonators placed at a finite distance from
65 each other along the duct axial direction. **In this work, we focus on ducts of the order of centimeters wide and of**
66 **constant cross section. The absorbers are flush mounted to the walls of the duct walls, so as not to change its cross**
67 **section. This feature is chosen to limit the perturbations of the air flow in the duct. The absorbers line part, or all, of**

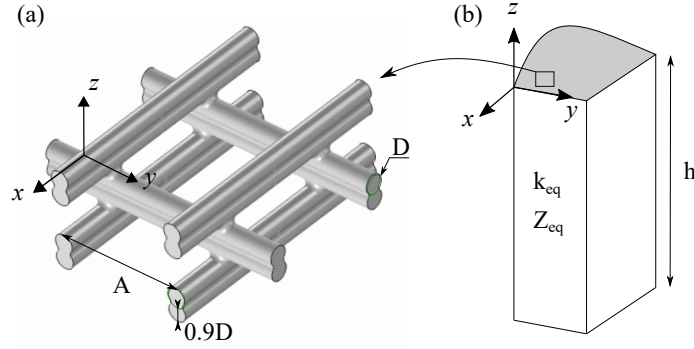


Figure 1: QWR filled with porous medium. (a) Diagram of the porous material microstructure named micro-lattice. The normal direction of the medium is z and is aligned to the normal direction of the QWRs. (b) QWR of arbitrary cross-section and height h .

the walls of the duct. Usually, the resonators composing the duct absorbers are HRs. We focus here on QWRs filled either with air or with a porous medium.

2.2. Porous medium

The porous medium filling the QWRs is a micro-lattice. The optimizations are performed considering this specific medium for efficiency purposes, but other media such as stochastic foams could have also been used. An idealized representation of the micro-lattice is depicted in Fig. 1 (a). This medium is fibrous, structured, periodic, quasi-isotropic and can be 3D printed. It is composed of a superposition of layers of parallel rods, each layer being orthogonal to the previous one. Each rod is a combination of two filaments of diameter D fused with one another in the vertical direction. The vertical, (O, z) , distance between the center of two successive filaments is $0.9D$. The horizontal distance between the center of two rods, i.e., the lattice constant, is named A . Both D and A can be tuned.

The optimized absorbers are made by additive manufacturing and more specifically by Fused Deposition Modeling (FDM) technique. The slicer software and printer used in this work are *Simplify3D* and *RAISE3D Pro2*, respectively. The extruded material is polylactic acid (PLA). A $200\ \mu\text{m}$ diameter nozzle is used to produce the micro-lattice. The manufacturing parameter that controls the lattice constant, A , is named "infill factor", $IF = 100/(AD)$, and is then inversely proportional to A .

2.3. QWRs filled with an anisotropic porous medium or air

The QWRs of the proposed absorbers are either filled with air or with the micro-lattice. This allows us to control the amount of losses introduced in each QWR.

2.3.1. Equivalent fluid modeling

To account for the acoustic losses occurring inside the QWRs, they are filled with equivalent fluids that are defined by an equivalent density ρ_{eq} and an equivalent bulk modulus K_{eq} , both being complex and frequency dependent. K_{eq} is a scalar and ρ_{eq} is a scalar if the filling medium is isotropic (e.g. air) or is a matrix if the filling medium is anisotropic (e.g. the quasi-isotropic micro-lattice).

The losses in a QWR filled with air arise from the viscous and thermal boundary layers along its walls. These losses are accounted for by the Stinson's model [20] considering an equivalent hydraulic radius to account for any type of QWR section. By contrast, the losses in a QWR filled with a porous medium arise mainly from the porous medium itself. Its corresponding equivalent fluid is described here by the well-known Johnson–Champoux–Allard–Lafarge (JCAL) model [21, 22]. It provides a frequency-dependent description of ρ_{eq} and K_{eq} by means of parameters depending on the porous medium micro-structure and called JCAL parameters. These are the porosity, ϕ , the tortuosity α_∞ , the viscous and thermal characteristic lengths, Λ and Λ' and the viscous and thermal static permeabilities q_0 and q'_0 . As can be seen in Fig. C.13 of the Appendix, ϕ , Λ , Λ' , q_0 and q'_0 decrease and α_∞ increases when the IF increases which indicates that the intrinsic losses and the resistivity of the micro-lattice increases when the IF increases (i.e., as the lattice constant decreases).

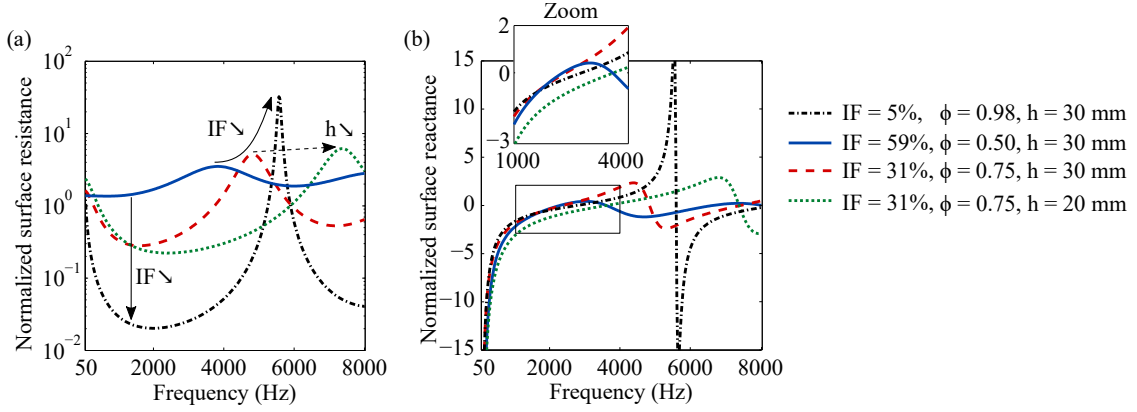


Figure 2: (color online) Normalized surface impedance of QWRs filled with homogeneous micro-lattices. The height of the QWRs is 30 mm or 20 mm. The diameter of the manufacturing nozzle is $200\mu\text{m}$ and the IF is equal to 5%, 31% or 59%, corresponding to a porosity of 0.98, 0.75 or 0.50, respectively. (a) Normalized surface resistance. (b) Normalized surface reactance.

101 During the porous absorber optimization, IF can take values comprised between 3% and 70%. In practice, it is only
 102 possible to manufacture integer percentage values of IF. As the micro-lattice fills QWRs of small lateral dimensions,
 103 for very low IF values, the lattice constant is close to the lateral size of the QWRs and the micro-lattice can no longer
 104 be considered as a homogeneous porous medium. For this reason the micro-lattice is replaced by air when the IF
 105 values are lower than 5%.

106 2.3.2. Surface impedance

107 *Expression of the surface impedance.* The QWRs composing the porous absorbers are forced to be of sufficiently
 108 small lateral dimensions compared to the acoustic wavelength to be considered as locally reacting resonators which
 109 behavior can be expressed through a surface impedance simplifying the waveguide propagation equations.

With an implicit time dependence $e^{i\omega t}$, and only accounting for normal propagation into the QWRs (locally reacting hypothesis), the surface impedance of a straight QWR filled with an homogeneous equivalent fluid is [22]

$$Z_s = -iZ_{eq} \cot(k_{eq}h), \quad (1)$$

110 where h is the height of the QWR, see Fig. 1 (b). k_{eq} and Z_{eq} are the equivalent fluid wave number and characteristic
 111 impedance, respectively, in the normal direction z . k_{eq} and Z_{eq} can be derived from ρ_{eq} and K_{eq} [23].

Propagation models that do not account for high order modes in the main waveguide, such as the TMM, require a length correction h_δ at the opening of the QWRs. A purely imaginary impedance is then added to Z_s and the QWR corrected surface impedance is

$$Z_s^c = Z_s + iZ_0k_0h_\delta, \quad (2)$$

112 in which $k_0 = \omega/c_0$ is the wave number of air in the waveguide and Z_0 is the characteristic impedance of the air in the
 113 waveguide. An expression of h_δ is in Ref. [24] for QWRs sections much smaller than $1/k_0$.

114 *Discussion about the surface impedance.* If a QWR is filled with air and its intrinsic losses are neglected, the surface
 115 impedance reduces to $Z_s = -iZ_0 \cot(k_0h)$. The only parameter of the QWR that controls Z_s is thus its height h . The
 116 surface resistance is zero for all frequencies and the surface reactance vanishes at the resonance frequencies of the
 117 QWR. Adding the length correction h_δ does not change the resistance and slightly shifts the null reactance frequencies.
 118 In practice, the intrinsic losses of a QWR are never exactly zero. As a consequence, its surface resistance is never
 119 exactly zero but its surface reactance still vanishes at the resonance frequencies.

120 If a QWR is filled with a micro-lattice, Z_s still depends on the QWR height h but also on the micro-lattice properties
 121 which vary with the filaments diameter and the lattice constant. In practice, the lattice constant and the height of the
 122 QWR can be easily tuned. For a given QWR height and 3D printer nozzle diameter, increasing the micro-lattice IF
 123 decreases the porous material lattice constant and thus increases its porosity and decreases its resistivity. This results

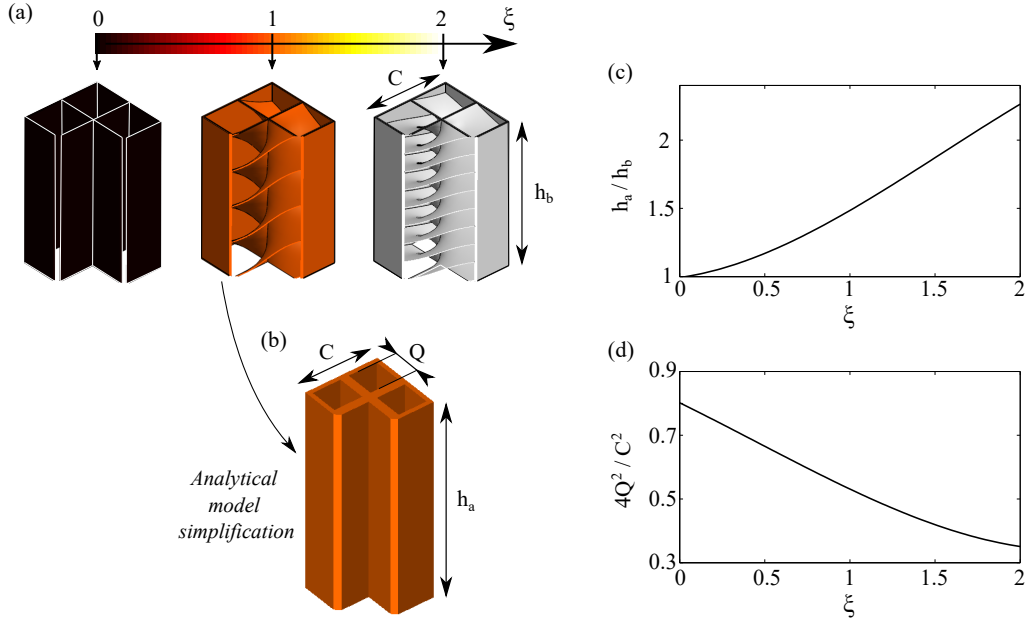


Figure 3: (a) Geometry of the structure of the folded QWRs with respect to ξ . (b) Simplified geometry of the $\xi = 1$ folded QWR. (c) Normalized height of the simplified folded QWRs in function of ξ . (d) Normalized section of the simplified folded QWRs in function of ξ .

124 in an increase of the quality factor of the porous QWR modes. Indeed, as depicted in Fig. 2, the peak of surface
 125 resistance and of surface reactance at the second quarter-wavelength frequency are sharper for lower IF values and
 126 the resistance is lower before and after the resistance peak. Moreover, the lower the IF, the higher the resonance
 127 frequency. Note that the surface resistance is never zero and its minimal value is obtained with the lowest authorized
 128 value of IF. In addition, as expected from Eq. (1), for a given IF (i.e., a given set of k_{eq} and Z_{eq}) decreasing the height
 129 h of the QWR increases the second quarter-wavelength frequency of the QWR and modifies its surface resistance and
 130 reactance. In particular, before the second quarter-wavelength frequency, the surface reactance is reduced (see zoom
 131 in Fig. 2 (b)).

132 Therefore, the surface impedance of a QWR filled with a homogeneous micro-lattice can be tuned by varying the
 133 IF of the micro-lattice and the QWR height. However, this tuning is limited by the fact that the surface impedance
 134 has peaks and valleys over the frequency range due to the QWR resonances and anti-resonances. Because of the
 135 intrinsic losses, a null surface resistance cannot be attained. The range of realizable impedance will determine how
 136 close the absorption coefficient of the absorber is to one. Around the resonance frequency of the QWR, where the
 137 QWR has the most impact on the absorber behavior, its surface reactance is mostly controlled by the QWR height
 138 and is approximately equal to $-Z_0 \cot(k_0 h)$. The minimum (resp. maximum) reachable surface resistance is obtained
 139 by minimizing (resp. maximizing) the QWR intrinsic losses. A very permeable (resp. resistive) porous material will
 140 lead to low (resp. high) losses. The lowest losses are obtained when filling the QWR with air.

141 2.3.3. Sub-wavelength folded QWRs

142 The term “sub-wavelength” usually indicates that an acoustic treatment of height h reaches perfect absorption at
 143 normal incidence when rigidly backed for a frequency corresponding to a wavelength in air λ that is much larger than
 144 $4h$. In practice, it is used to differentiate conventional porous treatments having a weak sub-wavelength behavior, i.e.,
 145 that cannot reach perfect absorption at normal incidence with $h \ll \lambda/4$, from other types of resonator based structures
 146 or resonant structures having a strong sub-wavelength behavior, such as treatments based on HRs. Treatments made
 147 of porous materials can also achieve sub-wavelength absorption, for instance by incorporating resonant inclusions [25,
 148 26, 27], optimizing a gradient of properties through their thickness [28, 29, 30] or folding their structure [31, 32, 33].
 149 To retain the essence of the term “sub-wavelength”, we suggest using it for ducted propagation to qualify an acoustic
 150 liner whose height is much smaller than $\lambda/4$.

To reach the sub-wavelength regime, we introduce folded QWRs. The interest of folding the QWRs is that their acoustic effective height h_a is then larger than the bulk QWRs height h_b . The sub-wavelength regime can then be reached since $h_b < h_a \approx \lambda/4$ while still filling the QWRs by simple homogeneous porous materials or air. Various folding strategies have been proposed in the literature, in particular for aerospace applications [34, 35]. The QWRs can be folded in a L- or U-shape or helically folded. The helically folded QWRs presented in Fig. 3 (a) are selected. Unlike L- and U-shape QWRs, the volume occupied by helically folded QWRs is not affected by the folding as these QWRs roll-up under themselves. Then, the optimization of a treatment made of a collection of helically folded QWRs is simple as it does not require topology optimization. These helically folded QWRs have been used in Ref. [33] to design a metaporous surface exhibiting broadband and sub-wavelength absorption at normal incidence. We now use them to design a sub-wavelength duct absorber optimized in a particular configuration in §5.3. The parameters governing the surface impedance of the helically folded QWRs are the number of revolutions ξ of their governing helix over h_b and the IF of their filling micro-lattices (if not replaced by air). The acoustic effective height and section of the helically folded QWRs is obtained through the comparison of numerical and analytical computations [33]. A ξ -folded QWR of height h_b and section C^2 is considered. First, the geometry of the QWR is simplified into an unfolded QWR of section C^2 made of a quartet of straight tubes of height h_a and section Q^2 as shown in Fig. 3 (b). Both the folded QWR and the simplified unfolded QWR are filled with an arbitrary equivalent fluid. Then, the reflection coefficient at normal incidence of the folded QWR is computed numerically. Finally, the h_a and Q parameters are selected so that the reflection coefficient of the simplified QWR computed analytically matches the numerically calculated impedance of the folded QWR as closely as possible. The analytical reflection coefficient at normal incidence of the unfolded QWR is given by

$$R_{QWR} = \frac{\frac{C}{4Q^2}Z_s^c - Z_0}{\frac{C}{4Q^2}Z_s^c + Z_0}, \quad (3)$$

with Z_0 being the characteristic impedance of the air and Z_s^c the corrected surface impedance of the straight tubes, Eq. (2). The variations of h_a and $4Q^2$ in function of ξ are presented in Fig. 3 (c, d).

3. Waveguide modeling

The acoustic behavior of a flush mounted asymmetric absorber is described by its transmission coefficient $T(f)$, its reflection coefficient defined for incident waves coming from upstream $R^+(f)$ and its reflection coefficient defined for incident waves coming from downstream $R^-(f)$. The absorption coefficient of the absorber is $\alpha^\pm(f) = 1 - |R^\pm(f)|^2 - |T(f)|^2$ and has a maximal value of 1 when the absorption is perfect, i.e., when there is no transmission and no reflection for a given incident wave direction.

This section introduces three methods to predict the acoustic behavior of asymmetric absorbers and to optimize them. The amplitude of the sound waves is low enough to consider linear propagation and there is no air flow in the waveguide.

The dimensions of the waveguide cross section are greater than or equal to 1 cm, and the optimization target frequency is always greater than 1 kHz. Then, the visco-thermal losses along the rigid walls of the waveguide are much smaller than those introduced by the designed absorbers. Moreover, they are uncoupled from the losses of the loaded resonators. This way, the predicted losses only arise from the absorbers.

3.1. Transfer Matrix Method

The Transfer Matrix Method (TMM) provides an approximate description of the acoustic behavior of a waveguide lined by an absorber [36]. The absorbers are formed of point resonators (i.e., with no axial extent) and plane waves are assumed to propagate along the waveguide. The possible evanescent coupling between the resonators is therefore neglected. While the TMM provides a 1D model that does not account for all the complexity of the problem at hand, it still captures the key features. This method is thus useful to derive fundamental design rules for the asymmetric absorber.

The problem considered by the TMM is outlined in Fig. 4: point resonators are connected to a propagation line representing the waveguide. The waveguide has a characteristic impedance $\tilde{Z}_0 = Z_0/S_w$ with Z_0 being the impedance of air with no losses (the losses are neglected in the waveguide) and S_w the cross-sectional area of the waveguide.

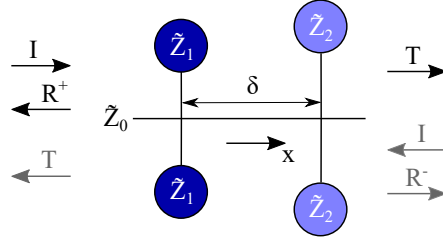


Figure 4: Schematic representation of a 1D asymmetric absorber composed of two different groups of localized resonators separated by a distance δ . Each group is composed of two resonators. The sound wave propagates in the x direction.

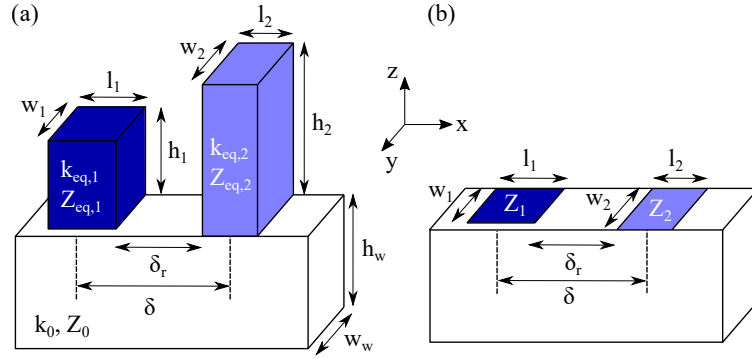


Figure 5: Diagram of a waveguide of rectangular section lined by two QWRs filled with equivalent fluids. (a) FEM representation. (b) MMT representation.

176 Each point resonator i is modeled by the surface impedance $\tilde{Z}_i = Z_i^c/S_i$ where Z_i^c is the surface impedance of the
 177 resonator (including a length correction) and S_i is the area of the resonator surface connected to the waveguide. As
 178 the model is one-dimensional, the respective areas S_w and S_i allow to represent the correct fluxes. Appendix A
 179 provides the complete details about the formulation of the TMM model.

180 3.2. Realistic waveguide models accounting for possible evanescent coupling

181 In this section, two more realistic models are presented to predict the absorption achieved by the porous asym-
 182 metric absorbers. These models are three-dimensional and include high-order duct modes. The possible couplings
 183 between resonators are therefore accounted for.

184 3.2.1. Finite element method

A finite element method (FEM) model of the lined waveguide is designed with COMSOL Multiphysics[®] and used
 for validation purposes. The QWRs are filled with an equivalent fluid and the waveguide is filled with air. The losses
 in the system only arise from the equivalent fluids. Fig. 5 (a) provides a schematic representation of this model for a
 waveguide lined with two straight QWRs filled with equivalent fluids. **The main equation solved by the FEM model
 is the generalized Helmholtz equation in the fluids (air or equivalent fluid),**

$$\frac{\omega^2}{K} p + \text{div}(\rho^{-1} \mathbf{grad} p) = 0, \quad (4)$$

where ρ^{-1} and K are the inverse of the density matrix and the bulk modulus of the fluid, respectively, both being
 complex and frequency dependent. Impervious walls are accounted by zero-flux boundary condition

$$\mathbf{grad} p \cdot \mathbf{n} = 0, \quad (5)$$

185 where \mathbf{n} is the normal vector to the walls.

186 The R^\pm and T coefficients of the absorber can be easily retrieved from the FEM simulations [37]. It involves
 187 projecting the pressure fields onto the modal basis of the rigid waveguide to obtain the modal amplitudes of the
 188 forward and backward waves and then computing R^\pm and T . Below the first cut-off frequency of the waveguide, the
 189 reflection and transmission coefficients of the absorber can be directly computed by the same technique as with a
 190 4-microphone impedance tube (see ISO 10534-2:1998).

191 3.2.2. Mode matching

192 The mode matching technique (MMT) described by Bi *et al.* [38] for cylindrical waveguides is adapted here to
 193 rectangular section waveguides.

194 The MMT considers a waveguide of constant section with impedance boundary conditions on the walls, see
 195 Fig. 5 (b). Unlike the FEM model, the full geometry of the QWRs is not directly represented as the resonators
 196 are replaced by their effective surface impedance. Rigid walls correspond to an infinite surface impedance. The
 197 surface impedance is piecewise continuous along the axial direction \mathbf{x} and can be continuous or continuous by part
 198 in the transverse directions. In each duct section, the acoustic fields are decomposed onto the modal basis of the
 199 hard-wall waveguide. Pressure and axial velocity are matched on the axial interfaces between two duct sections with
 200 different wall impedances. Since high-order duct modes are included, frequencies above the cut-off frequency of the
 201 waveguide can be considered. Note that the modeling technique does not require any length correction at the opening
 202 of the resonators because the coupling is operated via the high-order modes of the waveguides that are accounted for.
 203 See Appendix B for more details on this MMT model.

204 Unlike the TMM, the MMT does not allow to write simple expressions of optimal impedances but it considers the
 205 potential coupling between the resonators while delivering faster computation times than the FEM model. For this
 206 reason, the MMT is used to predict the behavior of the absorbers during their optimization.

207 4. Conditions for perfect absorption in asymmetric absorbers

208 In this section, the TMM model is used to derive conditions to achieve perfect absorption in waveguides with
 209 asymmetric absorbers. These conditions will be exploited in Sec. 5 to optimize asymmetric absorbers composed of
 210 QWRs but they are general enough to be applied to any types of resonators that can be considered as point resonators
 211 as a first approximation. The present analysis focuses on maximizing α^+ , i.e., minimizing $|R^+|^2 + |T|^2$. This problem
 212 is twofold since one has to suppress both the transmission and the reflection at the incident side.

213 4.1. Perfect absorption at a single frequency

214 4.1.1. Perfect absorption conditions

215 *Analytical expression of the perfect absorption conditions.* We start by considering a simple problem: an absorber
 216 made of two different point resonators of impedance \tilde{Z}_1 and \tilde{Z}_2 , respectively, and separated by a distance $\delta > 0$. This
 217 distance must be non-zero to produce an asymmetric absorber as shown in Fig. 4. Each resonator can be replicated
 218 and thus replaced by a group of $N \geq 1$ identical resonators located at the same axial position, for instance on multiple
 219 (N) walls lined by the same acoustic treatment.

The absorber transmission and reflection coefficients are

$$T = \frac{2\beta_1\beta_2 e^{-ik_0\delta}}{2\beta_1\beta_2 + \beta_1 + \beta_2 + i \sin(k_0\delta)e^{-ik_0\delta}}, \quad (6)$$

$$R^+ = -\frac{\beta_1 e^{-2ik_0\delta} + \beta_2 + i \sin(k_0\delta)e^{-ik_0\delta}}{2\beta_1\beta_2 + \beta_1 + \beta_2 + i \sin(k_0\delta)e^{-ik_0\delta}}, \quad (7)$$

$$R^- = -\frac{\beta_1 + \beta_2 e^{-2ik_0\delta} + i \sin(k_0\delta)e^{-ik_0\delta}}{2\beta_1\beta_2 + \beta_1 + \beta_2 + i \sin(k_0\delta)e^{-ik_0\delta}}, \quad (8)$$

220 with $\beta_1(f) = \tilde{Z}_1(f)/(N\tilde{Z}_0)$ and $\beta_2 = \tilde{Z}_2(f)/(N\tilde{Z}_0)$.

Due to the reciprocity of the [system composed of the waveguide lined by the absorber](#), the expression for T is
 symmetric with respect to β_1 and β_2 , i.e., the transmission coefficient of the absorber does not depend on the incident
 wave direction or on the ordering of the resonator groups: $T(\beta_1, \beta_2) = T(\beta_2, \beta_1)$. Either the resonator group 1 or

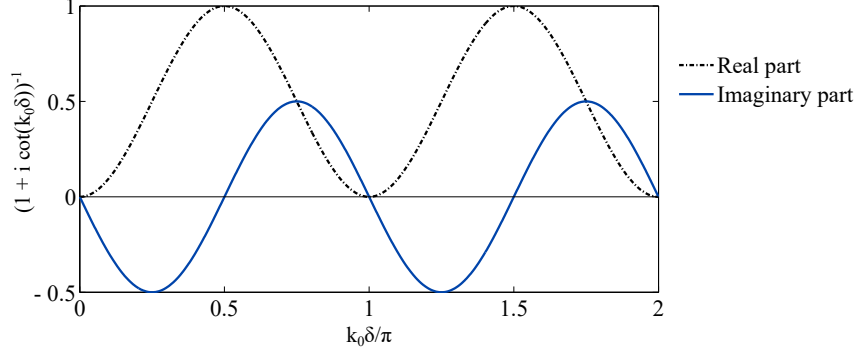


Figure 6: Real part (dash-dot line) and imaginary part (solid line) of the right-hand side of Eq. (12) with respect to non dimensional distance between resonators groups.

group 2 can be used to cancel the transmission. Here, the resonator group 2 is chosen to cancel T . To get zero transmission at a given frequency f , this second group should be designed such that

$$\beta_2 = \frac{\tilde{Z}_2(f)}{N\tilde{Z}_0} = 0 \quad \Leftrightarrow \quad Z_2^c(f) = 0. \quad (9)$$

In a less stringent form, a quasi no-transmission condition can be written as

$$\beta_2 = \frac{\tilde{Z}_2(f)}{N\tilde{Z}_0} \ll 1 \quad \Leftrightarrow \quad \frac{Z_2^c(f)}{Z_0} \ll \frac{NS_2}{S_w}. \quad (10)$$

Assuming that the resonator group 2 cancels the transmission at a given frequency f , the reflection coefficients of the absorber at that same frequency can be simplified in

$$R^+(f) = -\frac{\beta_1 e^{-ik_0\delta} + i \sin(k_0\delta)}{\beta_1 e^{+ik_0\delta} + i \sin(k_0\delta)}, \quad R^-(f) = -1, \quad \text{when } \beta_2(f) = 0. \quad (11)$$

On the one hand, the downstream reflection coefficient R^- is equal to -1 , meaning that the pressure wave is fully reflected by the resonator group 2. On the other hand, the upstream no-reflection condition $R^+ = 0$ can be obtained by canceling the numerator of R^+ , leading to

$$\beta_1(f) = \frac{\tilde{Z}_1(f)}{N\tilde{Z}_0} = \frac{1}{1 + i \cot(k_0\delta)} \quad \Leftrightarrow \quad \frac{Z_1^c(f)}{Z_0} = \frac{NS_1}{S_w} \frac{1}{1 + i \cot(k_0\delta)}. \quad (12)$$

221 *Discussion of the conditions for perfect absorption*. The surface reactance of a resonator vanishes at its resonances.
 222 If there are no inner losses in a resonator, its surface resistance will vanish at any frequency and the perfect no-
 223 transmission condition Eq. (9) can be satisfied at the resonance frequencies of the resonator. However, in practice, the
 224 inner losses of a resonator are never zero. Its surface resistance cannot be exactly equal to zero while a zero surface
 225 reactance is still reached at the resonance frequencies of the resonator. **The lowest transmission that can be achieved**
 226 **is limited by the surface resistance of the resonators and will depend on how well the Eq. (10) impedance condition is**
 227 **met. This way, for a given normalized surface impedance Z_2^c/Z_0 , the larger NS_2/S_w , the lower the transmission. This**
 228 **is the fundamental reason why reducing the transmission in narrow waveguides is easier than in wide waveguides.**
 229 **Note that duplicating the resonator group 2 and adjusting the new resonator group position can also greatly reduce the**
 230 **transmission, as described in Appendix A.3.1. Finally, while the resonator group 2 is responsible of canceling the**
 231 **transmission at a given frequency, the resonator group 1 also impacts the transmission at this frequency.**

232 The resonators impedance of resonator group 1 leading to perfect no-reflection, Eq. (12), is proportional to the ratio
 233 NS_1/S_w and is governed by the distance δ and more precisely the dimensionless variable $k_0\delta$ as illustrated in Fig. 6.

234 Moreover, Fabry-Perot interferences of the absorber are found whenever $k_0\delta = n\pi$ with $n \in \mathbb{N}$ (i.e., when $\delta = n\lambda/2$).
 235 These interferences imply $R^+ = -1$ for any value of Z_1^c : this particular spacing forbids high absorption [3]. Away from
 236 Fabry-Perot interferences, perfect no-reflection can be obtained as the target resistance is not null. The no-reflection
 237 condition can also be understood as an impedance matching between the characteristic impedance of the propagation
 238 line and the impedance at the x position of the resonator group 1 [12]: $(N\tilde{Z}_0)^{-1} = (iN\tilde{Z}_0 \tan(k_0\delta))^{-1} + (\tilde{Z}_1)^{-1}$.

239 **Perfect absorption consists in reflection and transmission coefficients exactly equal to zero. In practice, the trans-**
 240 **mission coefficient is never exactly equal to zero but can be extremely close to zero. The reflection coefficient can be**
 241 **exactly equal to zero. Then, perfect absorption is the objective of the optimization problem.**

242 The ratio between the area of the resonators surface connected to the propagation line NS_1 and NS_2 and the area of
 243 the propagation line cross-section S_w , is common to the quasi no-transmission and no-reflection conditions, Eqs. (10)
 244 and (12) and will appear in other expressions below. As it is an area ratio, it can be related to the leakage flux.

245 Merkel *et al.* [3], Jiménez *et al.* [5] and Long *et al.* [12] have designed asymmetric absorbers made of two HRs
 246 for perfect absorption at a single frequency by optimizing the absorbers parameters. The absorption is not predicted
 247 but explained *a posteriori* of the optimization. The pair of optimized HRs is found to be slightly detuned: the first
 248 resonance of resonator 2 f_2 such that $\Im(Z_2^c(f_2)) = 0$ occurs at slightly lower frequency than that of resonator 1, f_1 ,
 249 where \Im denotes the imaginary part of a complex number. The origin of this slight detuning can be explained by
 250 Eqs. (9) and (12). Resonator 2 makes the transmission vanish at its resonance frequency, f_2 , Eq. (9). To cancel the
 251 reflection, the resonator 1 surface reactance must take a small value, Eq. (12). The surface reactance of resonator 1
 252 is then low and, as the surface reactance of an HR or an homogeneously filled QWR monotonically increases until
 253 its first anti-resonance, $\Im(Z_1^c) = 0$ occurs at a frequency f_1 slightly different than f_2 . Resonators close to each other
 254 ($0 < k_0\delta < \pi/2$) implies that the optimal reactance for resonator 1 is negative at f_2 and thus $f_1 > f_2$. Conversely,
 255 $\pi/2 < k_0\delta < \pi$ leads to $f_2 > f_1$: the resonance of resonator 1 can occur at a lower frequency than that of resonator 2
 256 while still respecting the perfect no-reflection condition. Long *et al.* [11] have also shown that the HRs forming an
 257 asymmetric absorber decorated for perfect absorption can be tuned at the same resonance frequency. This particular
 258 configuration occurs for $k_0\delta = \pi/2$.

259 Thus, two groups of point resonators (each group can be composed of a single resonator) separated by a distance
 260 δ can be designed for perfect absorption, with the surface impedance of the resonators of group 2 following Eq. (10)
 261 and that of group 1 following Eq. (12), as long as $k_0\delta \neq n\pi$. To avoid designing unnecessary long absorbers, $k_0\delta < \pi$
 262 should be used.

263 4.1.2. Minimal absorber length

In many practical applications, the length of the absorber, l_A , must be as short as possible. Considering an absorber
 made of two different groups of resonators (see Fig. 4), if the surface impedance of the resonators can take any values
 at a given frequency, the length of the absorber $l_{AA} = \delta$, can be very small but has to remain greater than zero. In
 practice, the reachable values of impedance depend on the type of used resonator and on the resonator dimensions,
 as presented in Fig. 2 and discussed in §2.3.2 for QWRs filled with a micro-lattice. The no-transmission condition
 Eq. (9), or its approximation Eq. (10), does not depend on δ . For the group 2, it can be reached for instance by using a
 QWR filled with air and adjusting its height, or a HR filled with air and adjusting the dimensions of its neck and cavity.
 The no-reflection condition Eq. (12) depends on $k_0\delta$ and is usually harder to achieve because the surface resistance
 of the resonator must be of the order of NS_1/S_w , i.e., higher than zero but still very low. Its reactance is simpler to
 tune by varying the resonator height. Based on these considerations, at the targeted perfect absorption frequency, if
 the normalized surface resistance of the resonators composing the group 1 is written $\Re(Z_1^c/Z_0) = \Gamma$, where \Re denotes
 the real part of a complex number, and if the group 2 cancels the transmission, then achieving no-reflection requires

$$\Gamma = \frac{NS_1}{S_w} \sin^2(k_0\delta). \quad (13)$$

If $k_0\delta$ tends to zero, i.e., for low frequency and small δ , this expression can be approximated as

$$\delta = l_{A,min} = \frac{\lambda}{2\pi} \sqrt{\frac{\Gamma S_w}{NS_1}}, \quad \text{when } k_0\delta \ll 1. \quad (14)$$

264 This way, if the surface resistance of the group 1 resonators cannot be lower than a value Γ at the targeted perfect ab-
 265 sorption frequency, the minimum distance between the resonators groups, which is by consequence the minimum ab-

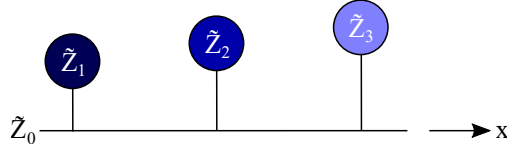


Figure 7: Schematic representation of a 1D asymmetric absorber composed of three different groups of resonators ($N = 1$) able to perfectly absorb two arbitrary frequencies. Each resonator is located at a different axial position x .

266 sorber length, is approximated by Eq. (14). Thus, the minimum absorber length is governed by the targeted frequency
 267 or its corresponding wavelength in air λ , the ratio between the duct section and the resonators section $S_w/(NS_1)$, as
 268 well as the minimal reachable normalized resistance Γ at this frequency. Note that, as shown in Appendix A.3.2,
 269 increasing the number of resonator groups can help decrease the minimal absorber length but the main features high-
 270 lighted in Eq. (14) are maintained.

271 In the literature, the length of duct liners, l_A , is often compared to the longest mostly absorbed wavelength in air, λ ,
 272 or $\lambda/4$ [4]. It does not allow to make a fair difficulty of design comparison between the absorbers because, in the view
 273 of Eq. (14), it is clear that the λ/δ ratio strongly depends on the area of the waveguide cross section, S_w , and that the
 274 optimal impedance values are strongly linked to resonators and waveguide areas ratio, Eqs. (10) and (12). Moreover,
 275 multiple detuned resonators can be located at the same axial position [12] underlying the importance of accounting
 276 for the area covered by all the resonators instead of the length of the absorber.

277 Instead of comparing l_A to λ , we suggest comparing l_A to the length $\lambda\sqrt{S_w/S_A}$, with S_A being the total area of the
 278 walls of the waveguide covered by the absorber, i.e., the volume of the absorber divided by its height. This expression
 279 combines three dimensions: the acoustic wavelength, λ , the area of the cross section of the propagation line, S_w , and
 280 the area of the walls of the waveguide covered by the absorber, S_A . The height of the absorber is not included in this
 281 expression because there is no physical link between this height and the minimal length of the absorber.

282 4.2. Conditions for perfect absorption at multiple close frequencies

283 A pair of resonators can target unidirectional perfect absorption at a given frequency with one implementing the
 284 quasi-no-transmission impedance, Eq. (10) and with the other one satisfying the no-reflection impedance, Eq. (12).
 285 Targeting perfect absorption over a broad frequency range requires to couple absorption peaks resulting from a com-
 286 bination of more than two resonators.

287 Long *et al.* obtained four absorption peaks ($\alpha^+ \approx 94\%$) using eight different HRs placed in two groups of four HRs
 288 and two close absorption peaks with one reaching 100% using three different HRs placed at different axial positions
 289 [12]. Long *et al.* obtained $\alpha^+ > 95\%$ between 280 and 431 Hz by combining twelve HRs placed in four groups of
 290 three HRs [11]. Note that, as long as at least two different resonators are placed at different axial positions to break
 291 the propagation symmetry, placing different resonators at the same axial position is not fundamentally different from
 292 spacing the different resonators provided they are not interacting, see Appendix A.4. In addition, Guo *et al.* obtained
 293 four absorption peaks using five different HRs having an optimized separation distance [39].

294 The most straightforward way to target unidirectional perfect absorption at F frequencies, thus forming an asym-
 295 metric absorber for perfect multiband or broadband absorption, consists in putting together F pairs of resonators, with
 296 each pair being responsible for a single frequency. However, less than F pairs of resonators, i.e., $2F$ resonators, can
 297 be used while still reaching the same level of absorption at F frequencies. In fact, in an ideal case where resonators
 298 could take any surface impedance value at any frequency, perfect absorption for F frequencies could be achieved by
 299 a single pair of such resonators. In practice, the impedance of passive resonators such as HRs or QWRs is difficult or
 300 impossible to tune at several given frequencies simultaneously since the frequency dependence of the impedance is
 301 prescribed by the underlying physical mechanisms. In particular, standard resonators such as HRs and QWRs filled
 302 with air can only fulfill the quasi-no-transmission Eq. (10) at their resonance frequencies. The transmission is also
 303 reduced around the resonance frequencies and the thinner the waveguide, the stronger and the broader the attenuation.
 304 If the frequencies targeted by the broadband asymmetric absorber are arbitrary and the width of the duct is of the order
 305 of centimeters, then there should be at least F resonators to cancel the transmission at F frequencies. A multi-order
 306 resonator, i.e., a resonator composed of multiple resonant elements [13], should be considered as O resonators with
 307 O being the resonator order. The no-reflection impedance condition Eq. (12), with δ being the distance between the

308 two resonators responsible for a given frequency, is slightly modified if other resonators are located between these
 309 two resonators. However, the necessity remains to reach a certain non-zero surface impedance that depends on $k_0\delta$ to
 310 cancel the reflection. It can be reached at multiple frequencies by a single resonator. Then, the number of resonators
 311 of the broadband asymmetric absorber lining a duct having a width of the order of centimeters should at least be equal
 312 to $F + 1$ and can be lower than $2F$. The absorber shown in Fig. 7 made of three resonators can thus target perfect
 313 absorption at two distinct frequencies.

314 Tuning the distances between the resonators and using a type of resonators whose surface impedance can be easily
 315 adjusted at several frequencies help in reducing the required number of resonators.

316 A minimum of $F + 1$ resonators is needed to reach unidirectional perfect absorption. To simultaneously reach
 317 perfect absorption from both sides of the system at the same frequency the absorber can be symmetrized around the
 318 last resonator. The absorber is then mirror symmetric and composed of $2F + 1$ resonators [3]. This number can be
 319 reduced in the best realistic scenario to $F + 2$ resonators because F resonators are required to cancel the transmission
 320 at F frequencies and 1 resonator is required to cancel the reflection in each direction for all frequencies. The resonator
 321 canceling R^+ is placed at the beginning of the absorber and the resonator canceling R^- is placed at the end of the
 322 absorber.

323 It is also important to note that every resonator impacts the reflection and transmission coefficients of the absorber
 324 at all frequencies. The increase of the number of resonators composing an absorber facilitates the obtaining of a high
 325 absorption and complicates the obtaining of the perfect no-reflexion.

326 5. Optimization in realistic conditions

327 Based on the TMM analysis presented in the previous section, it is possible to target perfect absorption at a single
 328 and at multiple frequencies by means of porous asymmetric absorbers. Moreover, the length of the absorbers could
 329 potentially be shorter than $\lambda/4$, provided the waveguide cross-section is not too large and its height is lower than
 330 $\lambda/4$ using folded QWRs. In this section, the porous absorbers are optimized using the more detailed MMT in order
 331 to show that the key results obtained with the TMM are still valid when possible coupling between the QWRs are
 332 accounted for.

333 5.1. Single frequency optimization

334 Absorbers made of two straight ($\xi = 0$) QWRs are optimized for three different configurations. The absorbers
 335 cover a single (x, y) wall of the waveguides. The lengths l_1 and l_2 of the QWRs are set to 5 mm. For simplicity, the
 336 cross section of the QWRs is square: the width of the QWRs is equal to their length, $w_1 = w_2 = 5$ mm. To keep
 337 the visco-thermal losses negligible in the main duct at the frequency range of interest, its width is $w_w = 10$ mm. In
 338 order to fully cover the (x, y) wall of the waveguide forming the absorber, the QWRs are duplicated in the transverse
 339 direction of the waveguide y . Their height h_1 and h_2 and the IF of their filling micro-lattice are optimized to maximize
 340 the absorption at $f = 2700$ Hz corresponding to a wavelength in air $\lambda = 127$ mm. For the MMT predictions, 25 and
 341 2 modes were used in the z and y directions, respectively. Maximizing the absorption coefficient α^+ is performed
 342 with the iterative Nelder-Mead algorithm [40] which is a heuristic direct search method implemented in the Matlab
 343 function *fminsearch*.

344 The first configuration aims to illustrate the validity of the TMM analysis to predict optimal impedances. It
 345 consists in a $h_w = 10$ mm waveguide and a long axial rigid section separating the QWRs of $\delta_r = 100$ mm. The
 346 optimized parameters and the resulting QWRs surface impedances, Z_1 and Z_2 , are summarized in Tab. 1 and $\alpha^+(f =$
 347 $2700 \text{ Hz}) = 100\%$. As expected, the second QWR is filled with air and has a sufficiently low surface impedance to
 348 strongly diminish the transmission, see Eq. (10). The TMM analysis led to the expression (12) for the impedance
 349 of the first resonator while the second resonator makes the transmission vanish. As summarized in Tab. 2, in the
 350 considered case, the TMM predicted an optimal impedance $Z_1^c/Z_0 = 0.39 + 0.20i$. For the MMT optimized absorber,
 351 the normalized surface impedance of the first QWR is close to this value with $Z_1/Z_0 = 0.40 + 0.18i$. Note that this
 352 last impedance value is not adjusted by a length correction because no length correction is needed in the MMT. In
 353 addition to the impact of the length correction, the small impedance difference with the TMM prediction may come
 354 from the fact that the second QWR does not perfectly makes the transmission vanish. Another explanation is that a
 355 coupling still occurs between the two resonators, despite the large distance δ_r between them, or that the resonators are
 356 too large to be perfectly modeled as point resonators.

357 The second and third configurations are more realistic: the distance between the two QWRs is reduced to $\delta_r =$
358 5 mm so that the total length of these absorbers is 15 mm = $\lambda/8.5$. The waveguide height is increased to either
359 $h_w = 20$ mm or $h_w = 40$ mm. Their optimized parameters and the resulting QWRs surface impedances are summarized
360 in Tab. 1 and their corresponding absorption coefficients are shown in Fig. 8 by the blue solid line and black dot-dashed
361 line, respectively. For the $h_w = 20$ mm waveguide the absorber is able to reach $\alpha^+(f = 2700 \text{ Hz}) = 100\%$. In contrast,
362 for the larger waveguide ($h_w = 40$ mm) the absorber achieves $\alpha^+(f = 2700 \text{ Hz}) = 90\%$ with a narrower absorption
363 peak. In these configurations, the optimal impedance of the first resonator predicted by the TMM, Eq. (12), and
364 summarized in Tab. 2 is less accurate than in the first configuration. This is because the resonators are close to each
365 other and thus most probably coupled. However, this equation still highlights that a large ratio S_w/S_1 requires low
366 Z_1 that might not be reached by the available porous material. It is the case for the $h_w = 40$ mm waveguide which
367 results in an optimized first QWR filled with air. The surface impedance of the first QWR is then solely governed by
368 the height of the QWR and its fixed (and thus not tuned) cross-section. Conversely, the first QWR optimal impedance
369 for the $h_w = 20$ mm waveguide can be attained by filling it with a tuned micro-lattice and $\alpha^+ = 100\%$ is then reached.
370 These optimizations based on a propagation model that includes the potential coupling between the resonators
371 confirm that, just as HRs, straight QWRs filled with homogeneous media (air or porous materials) can be used to
372 target perfect absorption in a ducted propagation problem. The QWR placed the farthest from the incident wave
373 is filled with air to obtain very low intrinsic losses. The transmission vanishes at the resonance frequency of the
374 resonator. The QWR closer to the incident wave is filled with a porous medium with a controlled pore size. This
375 way, its intrinsic losses are adjusted such that the absorber made of the two QWRs is impedance-matched with the
376 waveguide [12], leading to no reflection. As underlined by Eq. (14), the length of the resulting absorber can be
377 smaller than a quarter-wavelength in air at the fully absorbed frequency. These optimizations also show that the
378 simple expressions (9) and (12) for the optimal impedances, derived considering no coupling between the resonators,
379 give good trends of the actual optimal impedances and reliable predictions for large spacing between the resonators.
380 Finally, these optimizations highlight that the critical parameter controlling the minimal absorber length is not only
381 the wavelength but also the ratio between the waveguide section and the resonators section.

Table 1: Optimized parameters and corresponding normalized surface impedances of absorbers made of two QWRs of length $l_{1,2} = 5$ mm, separated by a rigid wall of length δ_r and lining a (x, y) wall of a waveguide of height h_w . The width of the QWRs is equal to that of the waveguide, $w_1 = w_2 = w_w = 5$ mm.

h_w (mm)	δ_r (mm)	IF_1 (%)	h_1 (mm)	Z_1/Z_0 (2700 Hz)	IF_2 (%)	h_2 (mm)	Z_2/Z_0 (2700 Hz)
10	100	33.4	27.8	$0.40 + 0.18i$	Air	30.0	$0.02 - 0.06i$
20	5	8.5	26.4	$0.05 - 0.20i$	Air	29.7	$0.02 - 0.08i$
40	5	Air	28.4	$0.02 - 0.15i$	Air	29.9	$0.02 - 0.08i$

Table 2: Optimal corrected surface impedances for perfect absorption at 2700 Hz, according to the TMM analysis assuming point resonators. The absorbers are made of two QWRs of length $l_{1,2} = 5$ mm, separated by a rigid wall of length δ_r and lining a (x, y) wall of a waveguide of height h_w . The width of the QWRs is equal to that of the waveguide, $w_1 = w_2 = w_w = 5$ mm.

h_w (mm)	δ_r (mm)	δ (mm)	Z_1^c/Z_0 , Eq. (12) (2700 Hz)	Z_2^c/Z_0 , Eq. (9) (2700 Hz)
10	100	105	$0.39 + 0.20i$	$0.00 + 0.00i$
20	5	10	$0.06 - 0.10i$	$0.00 + 0.00i$
40	5	10	$0.03 - 0.05i$	$0.00 + 0.00i$

382 5.2. Optimization for two close frequencies

383 To target perfect unidirectional absorption at two close frequencies $f = 2700$ Hz and 3000 Hz, an absorber com-
384 posed of three straight ($\xi = 0$) QWRs is optimized. A $h_w = 20$ mm and $w_w = 10$ mm waveguide is considered and
385 $w_1 = w_2 = w_3 = l_1 = l_2 = l_3 = 5$ mm. As in the configuration described in §5.1, the QWRs are duplicated in
386 the transverse direction of the waveguide. First, the parameters $h_1, h_2, h_3, IF_1, IF_2, IF_3$ of the absorber are optimized

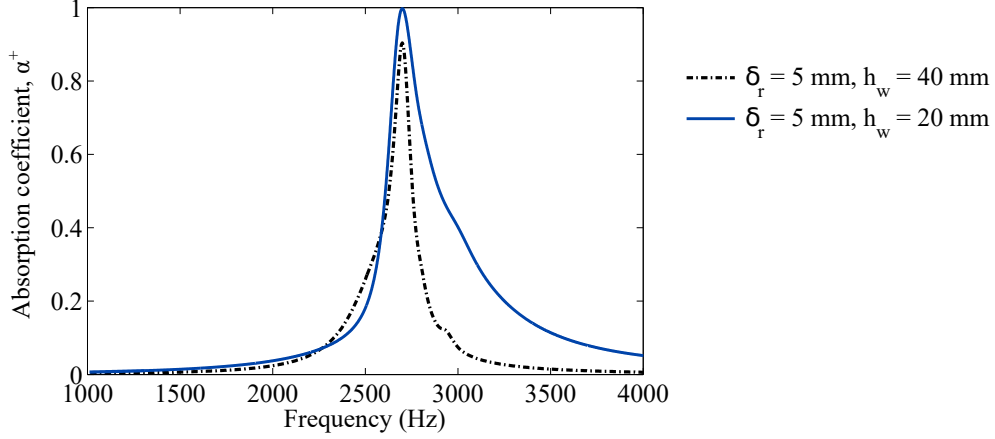


Figure 8: (color online) MMT predicted absorption coefficient of the optimized absorbers made of two QWRs of length $l_{1,2} = 5$ mm and separated by a rigid wall of length $\delta_r = 5$ mm. The waveguide height is $h_w = 20$ mm (blue solid line) or 40 mm (black dot dashed line).

Table 3: Optimized parameters of the absorbers for maximizing α^+ at 2700 Hz and 3000 Hz, made of three QWRs of length $l_{1,2,3} = 5$ mm, separated by δ_r rigid walls and lining a (x, y) wall of a $h_w = 20$ mm high waveguide. The width of the QWRs is equal to that of the waveguide, $w_1 = w_2 = w_3 = w_w = 5$ mm and $\bar{\alpha} = (\alpha(2700 \text{ Hz}) + \alpha(3000 \text{ Hz}))/2$.

δ_r (mm)	IF_1 (%)	h_1 (mm)	IF_2 (%)	h_2 (mm)	IF_3 (%)	h_3 (mm)	$\bar{\alpha}$
5.0	20	23.7	Air	30	Air	28.2	0.870
29.0	27.8	23.7	Air	25.6	Air	28.6	0.996
20.0	28.2	23.7	Air	28.6	Air	25.6	0.995

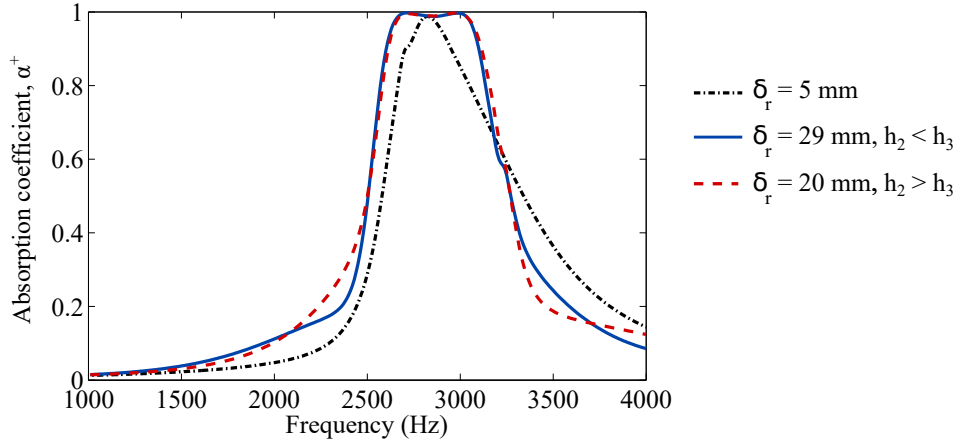


Figure 9: (color online) MMT predicted absorption coefficient of the optimized absorbers three QWRs of length $l_{1,2,3} = 5$ mm. The waveguide height is 20 mm and the QWRs are separated by rigid walls of length $\delta_r = 5$ mm (black dot dashed line), 20 mm (red dashed line) or 29 mm (blue solid line).

387 for a rigid separation distance $\delta_r = 5$ mm between each QWR. The optimized parameters are given in Tab. 3 and the
388 corresponding absorption coefficient is shown in Fig. 9 by a black dot-dashed line. The second and third QWRs are
389 filled with air and cancel the transmission at the two targeted frequencies. The first QWR is filled with a micro-lattice
390 and is not able to cancel perfectly the reflection at both frequencies. It means that with $\delta_r = 5$ mm, the optimal
391 impedance for the first QWR cannot be realized at these two frequencies with the considered filling porous medium.
392 This behavior was already observed in [12] where three HRs with fixed axial separation distance could not reach very
393 high absorption at two frequencies.

Table 4: Parameters of the folded QWRs composing the optimized ABSA of height $h_{ABSA} = 30$ mm.

Resonator	ξ	Layer 1	Layer 2	Simulation		Manufacture	
		height (mm)	height (mm)	$IF_1(\%)$	$IF_2(\%)$	$IF_1(\%)$	$IF_2(\%)$
1	0.27	24	6	30	29	26	25
2	0.65	9	21	28	25	34	21
3	1.57	9	21	15	9	12	6
4	0.50	30	-	10	-	7	-
5	0.41	30	-	Air	-	Air	-
6	0.86	30	-	8	-	5	-
7	1.01	30	-	8	-	5	-
8	1.19	30	-	7	-	4	-
9	1.33	30	-	9	-	6	-
10	1.91	30	-	Air	-	Air	-
11	1.59	30	-	Air	-	Air	-
12	1.73	30	-	Air	-	Air	-

In a second stage, the distance δ_r is also optimized, either forcing $h_2 < h_3$ or forcing $h_2 > h_3$. The optimized parameters are given in Tab. 3 and the corresponding absorption coefficients are depicted in Fig. 9 by the blue solid line and red dashed line, respectively. In both cases, considering δ_r as an additional optimization parameter allows to reach $\alpha^+ > 99.5\%$ at the two targeted frequencies. The surface impedance of the first porous QWR is matched to the optimal one. The axial position of the tallest QWR filled with air (i.e., the QWR making the transmission vanish at the lowest frequency) does not determine if $\alpha^+ = 100\%$ can be reached or not but it does influence the optimal δ_r and slightly influence the optimal IF_1 . The advantage of optimizing δ_r was already noted in [41, 39] where 5 HRs with optimized axial separation distance could reach very high absorption at 4 frequencies. However, in the mentioned work, the total length of the optimized absorber is larger than 1 m. Since the optimal surface impedance for the resonator responsible for canceling the transmission, Eq. (12), is $k_0\delta/\pi$ periodic, δ can be reduced as long as $\delta \bmod \pi/k_0$ is not modified and as long as the coupling between the resonators is moderate.

Asymmetric sound absorbers made of QWRs for perfect absorption at multiple frequencies can thus be designed. The analysis of the system by means of TMM highlights that the minimal number of resonators to fully absorb F arbitrary frequencies is $F + 1$, as was illustrated by the $\alpha^+ > 99.5\%$ at two frequencies obtained by three QWRs. No clear rule could be derived for the positioning of the resonators responsible for canceling the transmission in function of the frequency. A more advanced type of porous absorber is presented in the following section.

5.3. Broad target frequency bandwidth and sub-wavelength absorption

Finally, an Asymmetric Broad target frequency bandwidth Sub-wavelength Absorber (ABSA) made of folded ($\xi \geq 0$) QWRs filled with micro-lattices or air is optimized for perfect absorption. The proposed ABSA has a length $l_{ABSA} = 200$ mm, a width of 50 mm and is composed of folded QWRs of height $h_{ABSA} = 30$ mm. It lines the bottom wall of a waveguide of height $h_w = 40$ mm and width $w_w = 50$ mm, see Fig. 10 (a). Its open area ratio is then $h_w/(h_w + h_{ABSA}) = 0.57$. It is composed of 12 distinct and closely packed folded QWRs in the axial direction of the waveguide. These QWRs are duplicated 3 times along the width of the waveguide. After various numerical tests, it was found that a combination of 3 QWRs to make the reflection vanish and 9 QWRs to make the transmission vanish gave the most satisfactory results. The QWRs 1 to 3 are responsible for canceling the upstream reflection, R^+ . They are filled with bi-layer micro-lattices to provide more freedom to tune their surface impedance and thus realize the optimal surface impedance at multiple frequencies. The QWRs 4 to 12 are filled with a homogeneous micro-lattice or with air and are responsible for canceling the transmission. Obviously, QWRs 1 to 3 also have an impact on the transmission and QWRs 4 to 12 also have an impact on the reflection.

5.3.1. Optimization

The ABSA is optimized to maximize its absorption coefficient over the frequency range $f \in [1250; 2150]$ Hz, evenly discretized by 24 points. The optimization is performed in two steps due to the large number of optimization

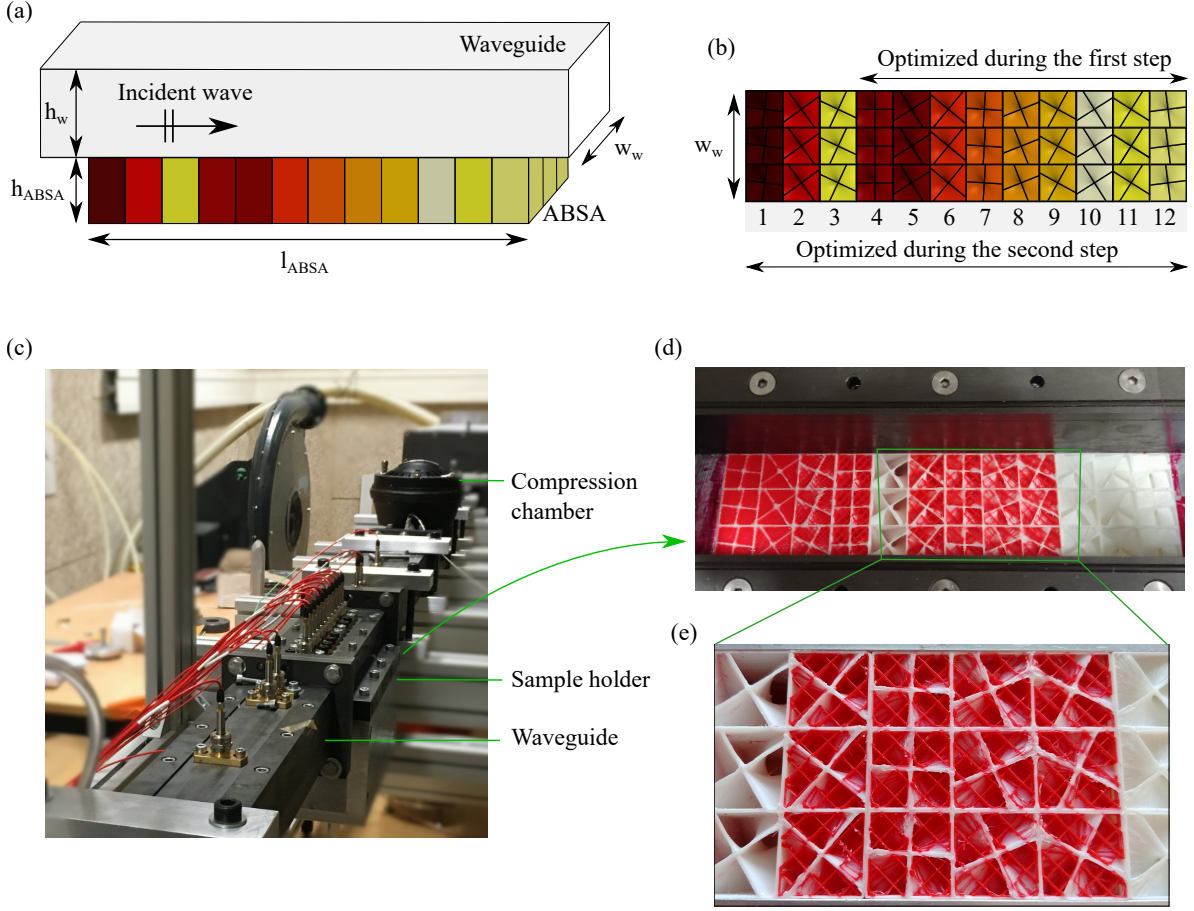


Figure 10: (color online) ABSA of length $l_{ABSA} = 200$ mm and height $h_{ABSA} = 30$ mm, flush mounted on a waveguide of height $h_w = 40$ mm and optimized for maximal average absorption over $f \in [1250; 2150]$ Hz. (a) Numerical perspective view of the ABSA lining the waveguide. (b) Numerical top view of the ABSA. (c) Picture of the duct test bench. (d) Picture of the 3D printed sample mounted in the duct test bench. (e) Picture of the 3D printed sample, close-up view.

426 parameters. The folded QWRs labeled 4 to 12 responsible for canceling the transmission are first considered and
 427 optimized by minimizing the cost function

$$J_T = \sum_f |T(f)|^2. \quad (15)$$

428 Then, in a second step, all the QWRs (1 to 12) are considered and optimized. The QWRs 4 to 12 are initialized
 429 with the values found during the first optimization step. The cost function J_{RT} for this second stage is

$$J_{RT} = \sum_f |R^+(f)|^2 + |T(f)|^2. \quad (16)$$

430 The optimized parameters for QWRs 4 to 12 are their ξ and IF. For QWRs 1 to 3, the optimized parameters are
 431 their ξ , the IF of the two layers named IF_1 and IF_2 and the height of layer 1, h_1 , layer 1 being the bottom layer. As
 432 the height of the folded QWRs is fixed and equal to h_{ABSA} , the height of layer 2 is $h_2 = h_{ABSA} - h_1$. The MMT is still
 433 used for the predictions of the acoustic behavior of the absorber during the optimization.

434 The main role of the resonators can be verified numerically after their optimization. For instance, only considering the
 435 optimized QWRs 4 to 12 leads to a low transmission and a non controlled reflection in the frequency range of interest.

436 The optimized parameters of the ABSA are summarized in Tab. 4 and the corresponding acoustic behavior is
 437 presented in Fig. 11 in terms of (a) the absorption coefficient α^+ , (b) the transmission loss $TL = -20 \log(|T|)$ and (c)
 438 $RL = -20 \log(|R^+|)$. The reflection and transmission coefficients are expressed in dB to highlight the differences at
 439 low absolute values. There is good agreement between the MMT predictions (dot-dashed black lines) and the FEM
 440 validation (solid blue lines). The small discrepancies in frequency and level can be ascribed to the simplified geometry
 441 of the folded porous QWRs when predicting their surface impedance used by the MMT. The FEM model predicts
 442 an absorption coefficient $\alpha^+ \in [98.8; 99.6]\%$ over the frequency range $f \in [1255; 2130]$ Hz with a mean value of
 443 $\bar{\alpha}^+ = 99.2\%$. The TL and the RL are comprised between 20 dB and 40 dB. The frequency $f = 1255$ Hz corresponds
 444 to a wavelength in air of $\lambda = 273$ mm. The absorber is therefore sub-wavelength with respect to its height with
 445 $h_{ABSA} = \lambda/9.1$. Its length is such that $l_{ABSA} = \lambda/1.4$. Following the discussion in §4.1.2, $l_{ABSA} = \lambda \sqrt{S_w/S_{ABSA}}/0.61$.

446 For frequencies lower than $f = 1255$ Hz both TL and α^+ drop towards 0. No resonator has a surface impedance
 447 close enough to zero required to cancel the transmission, Eq. (9). Conversely, the RL exhibits sharp peaks at null
 448 frequency and $f = 666$ Hz due to the 0th and 1st Fabry-Pérot interferences associated to the length 200 mm of the
 449 ABSA [3]. For frequencies above 2130 Hz, the TL and RL are low and start to oscillate for frequencies greater than
 450 3×1255 Hz (α^+ follows a similar trend). These oscillations are due to the resonator dynamics bringing their surface
 451 impedances close to optimal values for no transmission and no reflection because the impedances of QWRs take
 452 similar values around the QWRs first, third, fifth... quarter-wavelength frequencies, see Fig. 2.

453 **The designed and optimized ABSA is the first compact absorber to achieve $\alpha^+ > 99\%$ over a broad target fre-**
 454 **quency bandwidth while lining a waveguide of the order of centimeters wide. The absorber is compact in the sense**
 455 **that the resonators forming the ABSA are closely packed.**

456 5.3.2. Experimental validation

457 We now describe the manufacturing process of the optimized ABSA and its acoustic testing in a rectangular duct
 458 with grazing incident waves.

459 The folded QWRs composing the ABSA are 3D printed using Fused Deposition Modeling (FDM) technique. In
 460 addition to the $200 \mu\text{m}$ diameter nozzle responsible for the manufacturing of the micro-lattices (see §2.2), a $400 \mu\text{m}$
 461 diameter nozzle is responsible for the manufacturing of the rigid structures (vertical and helical walls). Four blocks
 462 of 3×3 folded QWRs are manufactured separately and assembled with vacuum grease on their external walls in the
 463 duct sample holder as depicted in Fig. 10 (c) and (d).

464 When the folded QWRs are filled with micro-lattices, the losses of the walls of the QWRs are neglected in all
 465 models. Experimentally, these additional losses can be approximately compensated by slightly decreasing the losses
 466 of the filling micro-lattices. To do so, their lattice constant is increased. More precisely, IF is decreased so that the
 467 micro-lattice porosity is increased by 0.03. This small value originates from the validation tests reported in [33]. The
 468 manufacturing parameters of the optimized ABSA are summarized in Tab. 4.

469 The manufactured ABSA is mounted in the wall of a rectangular duct of inner height $h_w = 40$ mm and width $w_w =$
 470 50 mm ended by an anechoic termination. This experimental facility and the associated measurement techniques have
 471 already been introduced and detailed in [42]. On each side of the sample, three microphones are placed and permit the
 472 measurements of the R^+ and T coefficients using a scattering matrix formalism. To that end, acoustic plane waves are
 473 generated by a compression chamber that can be mounted either upstream or downstream of the measurement section.
 474 The acoustic signal consists of a sine sweep going from 500 to 4000 Hz with a step of 2 Hz. The incident pressure
 475 wave amplitude is set to 90 dB at each frequency.

476 The measured acoustic behavior of the ABSA is represented in Fig. 11 by red dashed lines. There is a good
 477 correlation between the measured and the predicted results. The experimental performance is slightly lower than
 478 predicted in the targeted frequency range, and slightly higher for lower and higher frequencies. The small differences
 479 between the measured coefficients and those predicted by FEM may come mainly from the imperfection of the 3D
 480 printing, leading in particular to walls thicker than expected. The mounting of the sample in the duct can also induce
 481 some supplementary and undesired reflections. Nevertheless, the measured data are still very close to the predicted
 482 results, demonstrating the relevance of the proposed design. In particular, the manufactured ABSA sample is able to
 483 deliver an absorption coefficient $\alpha^+ \in [98.0; 99.5]\%$ in the frequency range $[1225; 2120]$ Hz, corresponding to a mean
 484 of value $\bar{\alpha}^+ = 98.7\%$, while the TL and RL values vary between 19 and 37 dB.

485 **This work considers no air flow in the main duct. Measurements of the ABSA with air flow are reported in [43].**

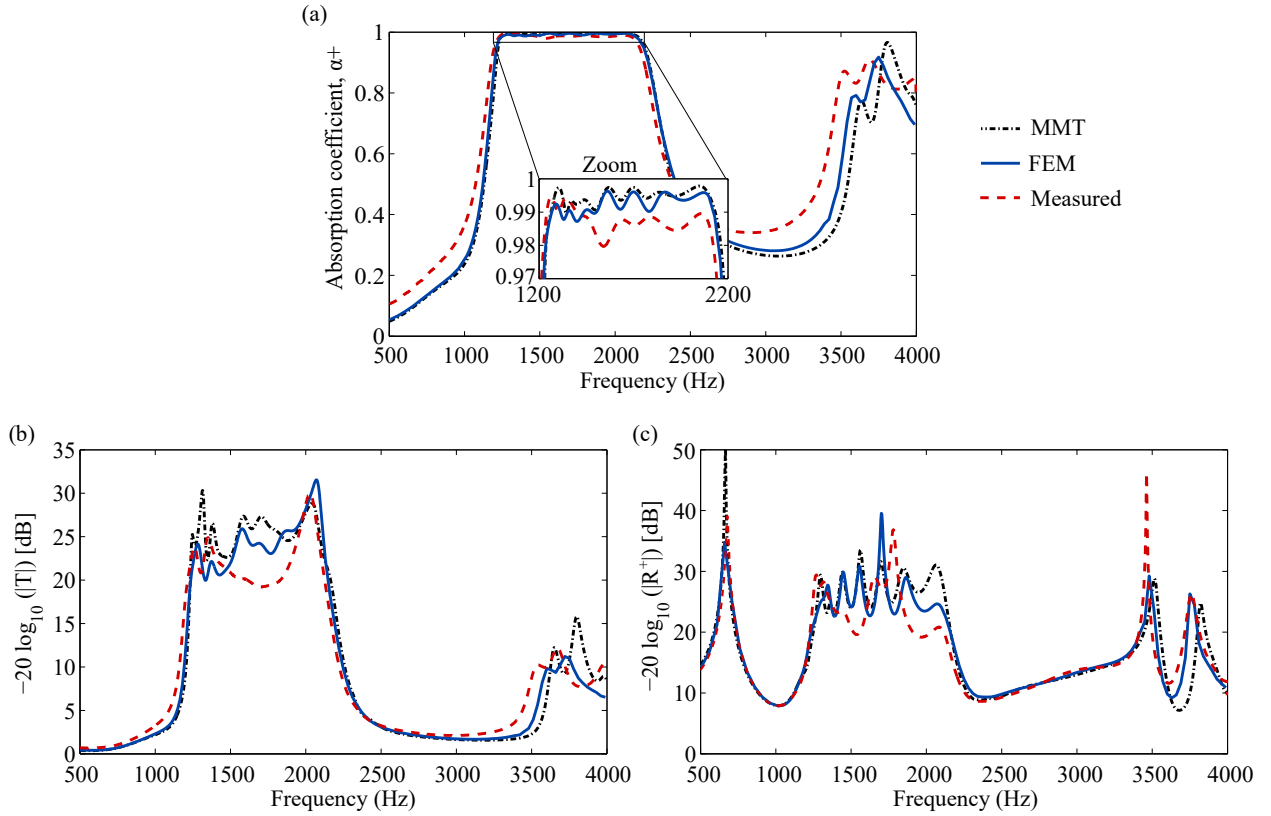


Figure 11: (color online) **Acoustic behavior of the optimized ABSA.** Simulations with the MMT (black dot-dashed lines) and with FEM (blue solid lines), measurements (red dashed lines). (a) Absorption coefficient α^+ . (b) Transmission loss $-20 \log_{10}(|T|)$. (c) $-20 \log_{10}(|R^+|)$.

486 6. Conclusions

487 This work explores perfect, **broad target frequency bandwidth** and sub-wavelength acoustic absorption in wave-
 488 guides of centimetric wide and constant cross section with porous asymmetric absorbers. A theoretical analysis based
 489 on the TMM provides insight and practical conditions for the understanding and design of such absorbers under the
 490 first cut-off frequency of the waveguide. Numerical simulations and experimental proof-of-concept illustrate that the
 491 theoretical results can be put into practice.

492 An asymmetric absorber is made of at least two resonators, the resonator farther from the incident wave cancels the
 493 transmission while the closer one implements an impedance match leading to no reflection. The no-transmission and
 494 no-reflection conditions are expressed in terms of the resonators surface impedances. The no-transmission condition
 495 is fulfilled by a zero impedance which can only be **quasi-perfectly** reached in practice due to the resonators losses.
 496 The no-reflection condition can be perfectly fulfilled by a non-zero impedance that depends on the spacing between
 497 the resonators and the target frequency. In both cases, the larger the ratio between the areas of the waveguide cross
 498 section and the resonator sections, the more difficult it is to reach the optimal impedances. In addition, scaling the
 499 system is not straightforward as the impedance of the resonators forming the absorber depend on their geometrical
 500 dimensions and the optimal no-reflection impedance depend on the spacing of the resonators.

501 The length of an asymmetric absorber, i.e., the distance in the propagation direction, optimized for perfect ab-
 502 sorption, can be strongly **smaller than a quarter of the wavelength in air** when using straight quarter-wavelength
 503 resonators filled with homogeneous media. Conversely, the height of an absorber made of such resonators cannot
 504 be sub-wavelength because no transmission is attained at resonant frequencies of the resonators. **The height of an**
 505 **asymmetric porous absorber can be rendered sub-wavelength**, for instance using folded quarter-wavelength resonators.

506 Broadband and very high absorption can be obtained by combining pairs of resonators responsible for absorbing

507 different close frequencies, thus forming a rippled plateau of absorption. The minimal number of resonators to cancel
 508 F arbitrary frequencies is $F + 1$, with F resonators canceling the transmission, located at the same or at different axial
 509 positions, and one resonator canceling the reflection placed at a different location than the resonators canceling the
 510 transmission.

511 Finally, a compact porous Asymmetric Broad target frequency bandwidth Sub-wavelength Absorber (ABSA)
 512 made of folded quarter-wavelength resonators is designed, optimized numerically, 3D printed and tested experimen-
 513 tally. A mean absorption coefficient of 99% is obtained experimentally over almost an octave and for frequencies
 514 smaller than the quarter-wavelength corresponding to the height of the absorber.

515 Acknowledgments

516 The authors acknowledge the financial support from the ANR industrial chair MACIA (ANR-16-CHIN-0002).
 517 They also acknowledge Safran Aircraft Engines and the Natural Sciences and Engineering Research Council of
 518 Canada (NSERC) for supporting and funding this research.

519 Appendix A. Transfer Matrix Method

520 Appendix A.1. Presentation of the method

A transfer matrix \mathbf{M} links the pressure p and axial flux v_x at two different positions along a transmission line [5].
 For instance, between x and $x + \delta$ we write

$$\begin{bmatrix} p \\ v_x \end{bmatrix}_x = \mathbf{M} \begin{bmatrix} p \\ v_x \end{bmatrix}_{x+\delta}. \quad (\text{A.1})$$

The transfer matrix of a rigid section of the propagation line of length δ and of cross-sectional area S_w is

$$\mathbf{M}_\delta = \begin{bmatrix} c & i\tilde{Z}_0 s \\ i s / \tilde{Z}_0 & c \end{bmatrix}, \quad (\text{A.2})$$

with $s = \sin(k_0\delta)$, $c = \cos(k_0\delta)$ and $\tilde{Z}_0 = Z_0/S_w$. The following is the transfer matrix for the i -th point resonator with a
 surface impedance Z_i^c (corrected by a length correction) and with a cross-sectional area connected to the propagation
 line S_i :

$$\mathbf{M}_i = \begin{bmatrix} 1 & 0 \\ 1/\tilde{Z}_i & 1 \end{bmatrix}, \quad (\text{A.3})$$

where $\tilde{Z}_i = Z_i^c/S_i$. The transfer matrix for the i -th resonator group composed of N_i identical point resonators with a
 surface impedance Z_i^c and with a cross-sectional area connected to the propagation line S_i is

$$\mathbf{M}_{g,i} = \begin{bmatrix} 1 & 0 \\ N_i/\tilde{Z}_i & 1 \end{bmatrix}, \quad (\text{A.4})$$

The transfer matrix of an absorber made of K resonator groups evenly spaced by a distance δ is

$$\mathbf{M}_a = \prod_{i=1}^K \mathbf{M}_{g,i} \mathbf{M}_\delta. \quad (\text{A.5})$$

The R^+ , R^- and T coefficients of the absorber are then given by

$$R^+ = \frac{\mathbf{M}_a(1, 1) - \mathbf{M}_a(2, 2) + \mathbf{M}_a(1, 2)/\tilde{Z}_0 - \mathbf{M}_a(2, 1)\tilde{Z}_0}{\mathbf{M}_a(1, 1) + \mathbf{M}_a(2, 2) + \mathbf{M}_a(1, 2)/\tilde{Z}_0 + \mathbf{M}_a(2, 1)\tilde{Z}_0}, \quad (\text{A.6})$$

$$R^- = \frac{-\mathbf{M}_a(1, 1) + \mathbf{M}_a(2, 2) + \mathbf{M}_a(1, 2)/\tilde{Z}_0 - \mathbf{M}_a(2, 1)\tilde{Z}_0}{\mathbf{M}_a(1, 1) + \mathbf{M}_a(2, 2) + \mathbf{M}_a(1, 2)/\tilde{Z}_0 + \mathbf{M}_a(2, 1)\tilde{Z}_0}, \quad (\text{A.7})$$

$$T = \frac{2e^{ik_0K\delta}}{\mathbf{M}_a(1, 1) + \mathbf{M}_a(2, 2) + \mathbf{M}_a(1, 2)/\tilde{Z}_0 + \mathbf{M}_a(2, 1)\tilde{Z}_0}. \quad (\text{A.8})$$

521 *Appendix A.2. Absorber made of two resonator groups*

We consider the case of an absorber composed of two distinct resonator groups separated by a distance δ . Each group is composed of N resonators. The transfer matrix corresponding to these two groups of resonators is

$$\mathbf{M}_a = \mathbf{M}_1 \mathbf{M}_\delta \mathbf{M}_2 = \begin{bmatrix} c + is \frac{N\tilde{Z}_0}{\tilde{Z}_2} & i\tilde{Z}_0 s \\ \frac{cN}{\tilde{Z}_2} + \frac{cN}{\tilde{Z}_1} + \frac{isN^2\tilde{Z}_0}{\tilde{Z}_1\tilde{Z}_2} + \frac{is}{\tilde{Z}_0} & c + is \frac{N\tilde{Z}_0}{\tilde{Z}_1} \end{bmatrix}. \quad (\text{A.9})$$

522 *Appendix A.3. Absorber made of several resonator groups, with some of them being identical*

523 *Appendix A.3.1. Two identical resonator groups to make the transmission vanish*

We consider the case where the two groups of resonators of an absorber are identical and composed of N identical resonators. The groups are separated by a distance δ and are used to make the transmission vanish. The transfer matrix corresponding to these two resonator groups is

$$\mathbf{M}_a = \mathbf{M}_2 \mathbf{M}_\delta \mathbf{M}_2 = \begin{bmatrix} c + is \frac{N\tilde{Z}_0}{\tilde{Z}_2} & i\tilde{Z}_0 s \\ \frac{2cN}{\tilde{Z}_2} + \frac{isN^2\tilde{Z}_0}{\tilde{Z}_2^2} + \frac{is}{\tilde{Z}_0} & c + is \frac{N\tilde{Z}_0}{\tilde{Z}_2} \end{bmatrix}. \quad (\text{A.10})$$

The corresponding transmission coefficient is

$$T = 2e^{ik_0\delta} \left[c(2 + 2\beta_2^{-1}) + is(2 + 2\beta_2^{-1} + \beta_2^{-2}) \right]^{-1}. \quad (\text{A.11})$$

524 Therefore, if $\beta_2 < 1$ the transmission can be much lower than when using a single resonator group to cancel the
 525 transmission, Eq. (10). This is due to the term with the square of β_2 in the expression for T for two identical resonator
 526 groups, Eq. (A.11).

527 *Appendix A.3.2. One resonator group to cancel the transmission and two identical resonator groups to cancel the
 528 reflection*

We consider an absorber that is composed of three resonator groups. Each group is formed of N identical resonators. The first and second resonator groups are identical and different than the third resonator group. The transfer matrix of this absorber is

$$\mathbf{M}_a = \mathbf{M}_1 \mathbf{M}_\delta \mathbf{M}_1 \mathbf{M}_\delta \mathbf{M}_3. \quad (\text{A.12})$$

If the third resonator group is designed to make the transmission vanish, $Z_3/Z_0 = 0$, a new target impedance for the resonator group responsible for making the reflection R^+ vanish can be written as

$$\frac{Z_1^{c,\pm}}{Z_0} = \frac{NS_1}{S_w} \frac{2}{1 + 3i \cot(k_0\delta) \pm \sqrt{\Delta}}, \quad \text{with } \sin(k_0\delta) \neq 0 \text{ and } \Delta = (1 + 3i \cot(k_0\delta))^2 + 4 \frac{e^{-2ik_0\delta}}{\sin^2(k_0\delta)}. \quad (\text{A.13})$$

529 The value $k_0\delta = n\pi$ still implies $R^+ = -1$ while $k_0\delta = (0.27 + n/2)\pi$ leads to the highest optimal resistance equal
 530 to $1.5NS_1/S_w$.

An absorber made of 3 resonator groups has a length of 2δ . For small values of $k_0\delta$, and with $\Re(Z_1^{c,+}/Z_0) = \Gamma$ the lowest reachable resistance, numerically we find that the minimum absorber length for perfect absorption is

$$l_{A,\min} = 2\delta = \frac{0.9}{k_0} \sqrt{\frac{\Gamma S_w}{NS_1}}, \quad \text{when } k_0\delta \ll 1. \quad (\text{A.14})$$

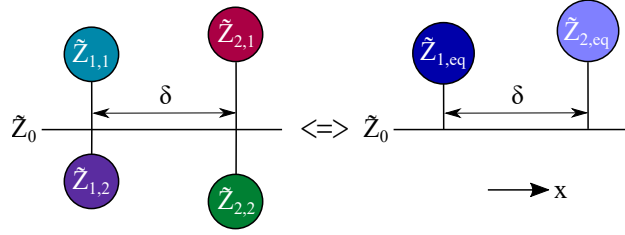


Figure A.12: Schematic representation of a 1D asymmetric absorber composed of two pairs of different resonators. Each pair is composed of two different resonators located at a given x position. The pairs are separated by a distance δ .

531 Appendix A.4. Multiple resonators at the same axial position

With the TMM, point resonators located at the same axial position x_i form a parallel circuit. If the resonators are independent, i.e., if they do not interact, the resonators $i, 1$ to i, N can be represented by a single equivalent resonator of surface impedance $\tilde{Z}_{i,eq}$ given by

$$\frac{1}{\tilde{Z}_{i,eq}} = \sum_{j=1}^N \frac{1}{\tilde{Z}_{i,j}}. \quad (\text{A.15})$$

532 If the resonators at a given position are identical ($\tilde{Z}_{i,1} = \tilde{Z}_{i,2} = \dots = \tilde{Z}_i$) then the equivalent surface impedance
 533 is simply $\tilde{Z}_{i,eq} = \tilde{Z}_i/N$. The group of identical resonators is equivalent to a resonator of lower impedance. This
 534 configuration was considered to derive the no-transmission condition, Eq. (10), the no-reflection condition Eq. (12)
 535 and the minimal absorber length Eq. (14). Everything goes as if the area S_w of the cross-section of the propagation
 536 line was divided by the number N of resonators at the same position, or as if the areas of the resonators connected to
 537 the propagation line, S_i , was multiplied by N . Identical resonators can be assembled in groups composed of different
 538 number of resonators and form an absorber that can be optimized for perfect absorption [13] as each group has a
 539 different equivalent impedance.

540 As schematized in Fig. A.12, if the resonators at a given position are different, i.e., $\tilde{Z}_{i,1} \neq \tilde{Z}_{i,2} \dots$ the equivalent
 541 surface impedance is a combination of their respective impedances.

542 Combining different resonators at a given axial position is useful to obtain an equivalent impedance having a
 543 frequency dependence that could not be realized by a single resonator [12]. For instance, if $\tilde{Z}_{i,1}(f_1) \approx 0$ and $\tilde{Z}_{i,2}(f_2) \approx$
 544 0 , then $\tilde{Z}_{i,eq} \approx 0$ at f_1 and f_2 and $T \approx 0$ at f_1 and f_2 which could not be achieved with identical resonators (unless f_2 is
 545 a multiple of f_1). Note that to maximize the cancellation of the transmission at a single frequency, the best approach
 546 is to combine identical resonators having a zero reactance and the minimal resistance allowed by the considered type
 547 of resonator, see Eq. (10). Placing different resonators at the same axial position rather than at different positions does
 548 not reduce the number of resonators required for perfect absorption at F frequencies but may help in reducing the
 549 length of a broadband absorber.

550 Appendix B. Mode matching technique

551 This Appendix presents the MMT equations used in this work. They are adapted from the work of Bi *et al.*[38],
 552 considering cylindrical waveguides, to rectangular cross-section waveguides. Only the specificities of rectangular
 553 cross-section waveguides are detailed hereafter.

555 Appendix B.1. Complete orthonormal basis

In the hard walled regions of the waveguide, the modal transverse wave numbers in the y and z directions of order $n \in \mathbb{N}^+$ and $m \in \mathbb{N}^+$, respectively, are

$$k_{yn} = \frac{n\pi}{d_w}, \quad k_{zm} = \frac{m\pi}{h_w}. \quad (\text{B.1})$$

The modal wave number in the direction of propagation x is

$$k_{xmn} = \sqrt{k_0^2 - k_{yn}^2 - k_{zm}^2}, \quad \text{Im}(k_{xmn}) \leq 0. \quad (\text{B.2})$$

A complete orthonormal basis of the hard walled waveguide region is the column vector ψ composed of the ψ_{mn} elements defined as

$$\psi_{mn}(y, z) = \cos(k_{yn}y) \cos(k_{zm}z) \sqrt{\frac{\epsilon_m \epsilon_n}{h_w d_w}}, \quad (\text{B.3})$$

556 with $\epsilon_0 = 1$, $\epsilon_M = 2 \forall M \neq 0$.

557

This way, the modal expression of the pressure field is

$$p(x, y, z) = \sum_m \sum_n (P_{mn}^+(x) + P_{mn}^-(x)) \psi_{mn}(y, z), \quad (\text{B.4})$$

$$p(x, y, z) = {}^t \psi (\mathbf{P}^+ + \mathbf{P}^-), \quad (\text{B.5})$$

$$p(x, y, z) = {}^t \psi \mathbf{P}, \quad (\text{B.6})$$

558 with \mathbf{P} , \mathbf{P}^+ , and \mathbf{P}^- column vectors of same dimension than ψ , being the modal amplitudes of the pressure field, of the
559 forward pressure field and of the backward pressure field, respectively. The transpose operator is noted t .

560

561 *Appendix B.2. Projection of p , $\partial p / \partial x$ and $\Delta_{\perp}(p)$ over the hard walled waveguide modal basis*

The projection of p and $\partial p / \partial x$ on the ψ basis read, respectively

$$\int_{y=0}^{d_w} \int_{z=0}^{h_w} p \psi_{mn} dy dz = P_{mn}, \quad (\text{B.7})$$

$$\int_{y=0}^{d_w} \int_{z=0}^{h_w} \frac{\partial p}{\partial z} \psi_{mn} dy dz = P'_{mn}. \quad (\text{B.8})$$

The projection of $\Delta_{\perp}(p)$ with $\Delta_{\perp} = \partial^2 / \partial y^2 + \partial^2 / \partial z^2$, on the ψ basis reads, according to the Green's second identity:

$$\int_{y=0}^{d_w} \int_{z=0}^{h_w} \Delta_{\perp}(p) \psi_{mn} dy dz = \int_y \int_z p \Delta_{\perp}(\psi_{mn}) dy dz + \int_{\Gamma} \psi_{mn} \mathbf{grad}_r(p) - p \mathbf{grad}_r(\psi_{mn}) d\Gamma, \quad (\text{B.9})$$

562 with Γ the edge of the waveguide at a position x .

563

As $\Delta_{\perp}(\psi_{mn}) = -(k_{yn}^2 + k_{zm}^2) \psi_{mn}$, then

$$\int_{y=0}^{d_w} \int_{z=0}^{h_w} p \Delta_{\perp}(\psi_{mn}) dy dz = -(k_{yn}^2 + k_{zm}^2) P_{mn}. \quad (\text{B.10})$$

The edges terms are

$$\int_{\Gamma} p \mathbf{grad}_r(\psi_{mn}) d\Gamma = 0, \quad (\text{B.11})$$

and, considering that all the waveguide walls can be lined by a treatment,

$$\int_{\Gamma} \mathbf{grad}_r(p) \psi d\Gamma = -i\omega \rho_0 \mathbf{PC}(x), \quad (\text{B.12})$$

with

$$\mathbf{C}(x) = \int_{y=0}^{d_w} \left(\frac{1}{Z_s(x, y, 0)} \psi(z=0) {}^t \psi(z=0) + \frac{1}{Z_s(x, y, h_w)} \psi(z=h_w) {}^t \psi(z=h_w) \right) dy \quad (\text{B.13})$$

$$+ \int_{z=0}^{h_w} \left(\frac{1}{Z_s(x, 0, z)} \psi(y=0) {}^t \psi(y=0) + \frac{1}{Z_s(x, d_w, z)} \psi(y=d_w) {}^t \psi(y=d_w) \right) dz. \quad (\text{B.14})$$

564 and Z_s the surface impedance at a given waveguide wall location.

565 *Appendix B.3. Modal propagation equation*

Then, the pressure modal differential equation in a portion of the waveguide of constant axial surface impedance is

$$\mathbf{P}''(x) + \mathbf{A}(x)\mathbf{P}(x) = \mathbf{0}, \quad (\text{B.15})$$

with \mathbf{A} a picewise constant tensor along x that reads

$$\mathbf{A}(x) = -\mathbf{L} + \frac{\omega^2 \rho_0}{K_0} \mathbf{I} - i\omega \rho_0 \mathbf{C}(x). \quad (\text{B.16})$$

566 and \mathbf{L} a diagonal matrix with $k_{ym}^2 + k_{zm}^2$ on the diagonal.

567 **Appendix C. JCAL parameters of the considered micro-lattice**

568 The variation of the porosity, ϕ , the tortuosity, α_∞ , the viscous and thermal characteristic lengths, Λ and Λ' and
 569 the viscous and thermal static permeabilities q_0 and q'_0 of the considered micro-lattice in function of IF are presented
 570 in Fig. C.13. These variations are obtained through the inverse characterization of several homogeneous samples
 571 characterized in an impedance tube [44]. As the micro-lattice is a quasi-isotropic medium, the value of α_∞ , Λ , q_0
 572 depend on the wave propagation direction (in-plane or out-of-plane).

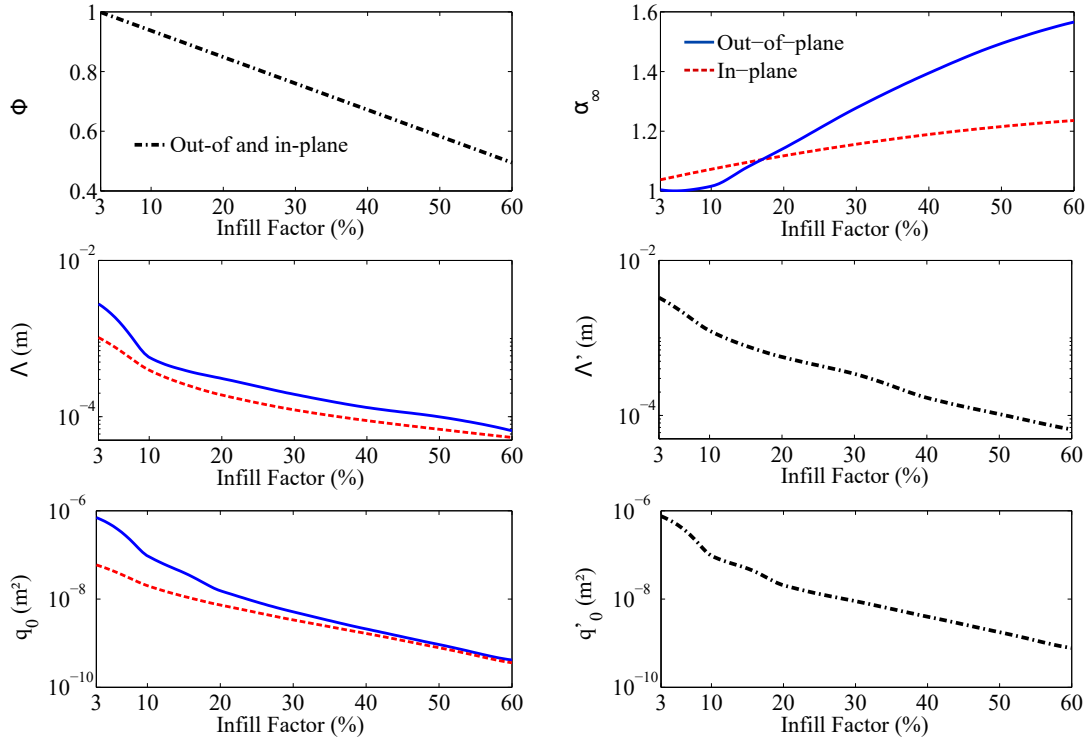


Figure C.13: Variation of the JCAL parameters of the micro-lattice in function of its Infill Factor (IF) obtained from an inverse characterization process based on impedance tube measurements.

573 **References**

- 574 [1] Potente, Daniel, General design principles for an automotive muffler, Busselton, 2005.
- 575 [2] J.-P. Dalmont, E. Portier, Optimisation of anechoic duct termination using line theory 117 141–144. doi:10.1016/j.apacoust.2016.
576 10.024.
577 URL <https://linkinghub.elsevier.com/retrieve/pii/S0003682X16304054>
- 578 [3] A. Merkel, G. Theocharis, O. Richoux, V. Romero-García, V. Pagneux, Control of acoustic absorption in one-dimensional scattering by
579 resonant scatterers, Applied Physics Letters 107 (24) (2015) 244102. doi:10.1063/1.4938121.
580 URL <http://aip.scitation.org/doi/10.1063/1.4938121>
- 581 [4] S. Kumar, H. P. Lee, Recent advances in acoustic metamaterials for simultaneous sound attenuation and air ventilation performances (2020)
582 25.
- 583 [5] N. Jiménez, V. Romero-García, V. Pagneux, J.-P. Groby, Rainbow-trapping absorbers: Broadband, perfect and asymmetric sound absorption
584 by subwavelength panels for transmission problems, Scientific Reports 7 (1) (2017) 13595. doi:10.1038/s41598-017-13706-4.
- 585 [6] N. Jiménez, W. Huang, V. Romero-García, V. Pagneux, J.-P. Groby, Ultra-thin metamaterial for perfect and quasi-omnidirectional sound
586 absorption, Applied Physics Letters 109 (12) (2016) 121902. doi:10.1063/1.4962328.
- 587 [7] X. Xiang, X. Wu, X. Li, P. Wu, H. He, Q. Mu, S. Wang, Y. Huang, W. Wen, Ultra-open ventilated metamaterial absorbers for sound-silencing
588 applications in environment with free air flows, Extreme Mechanics Letters 39 (2020) 100786. doi:10.1016/j.eml.2020.100786.
589 URL <https://linkinghub.elsevier.com/retrieve/pii/S2352431620301115>
- 590 [8] M. Yang, C. Meng, C. Fu, Y. Li, Z. Yang, P. Sheng, Subwavelength total acoustic absorption with degenerate resonators, Applied Physics
591 Letters 107 (10) (2015) 104104. doi:10.1063/1.4930944.
592 URL <http://aip.scitation.org/doi/10.1063/1.4930944>
- 593 [9] V. Romero-García, N. Jiménez, J.-P. Groby, A. Merkel, V. Tournat, G. Theocharis, O. Richoux, V. Pagneux, Perfect Absorption in Mirror-
594 Symmetric Acoustic Metascreens, Physical Review Applied 14 (5) (2020) 054055. doi:10.1103/PhysRevApplied.14.054055.
595 URL <https://link.aps.org/doi/10.1103/PhysRevApplied.14.054055>
- 596 [10] P. Wei, C. Croënne, S. Tak Chu, J. Li, Symmetrical and anti-symmetrical coherent perfect absorption for acoustic waves, Applied Physics
597 Letters 104 (12) (2014) 121902. doi:10.1063/1.4869462.
598 URL <http://aip.scitation.org/doi/10.1063/1.4869462>
- 599 [11] H. Long, C. Liu, C. Shao, Y. Cheng, J. Tao, X. Qiu, X. Liu, Tunable and broadband asymmetric sound absorptions with coupling of acoustic
600 bright and dark modes 479 115371. doi:10.1016/j.jsv.2020.115371.
- 601 [12] H. Long, Y. Cheng, X. Liu, Asymmetric absorber with multiband and broadband for low-frequency sound, Appl. Phys. Lett. (2017) 6.
- 602 [13] H. Long, Y. Cheng, X. Liu, Reconfigurable sound anomalous absorptions in transparent waveguide with modularized multi-order helmholtz
603 resonator 8 (1) 15678. doi:10.1038/s41598-018-34117-z.
- 604 [14] E. R. Fotsing, A. Dubourg, A. Ross, J. Mardjono, Acoustic properties of periodic micro-structures obtained by additive manufacturing,
605 Applied Acoustics 148 (2019) 322–331. doi:10.1016/j.apacoust.2018.12.030.
- 606 [15] S. Deshmukh, H. Ronge, S. Ramamoorthy, Design of periodic foam structures for acoustic applications: Concept, parametric study and
607 experimental validation, Materials & Design 175 (2019) 107830. doi:10.1016/j.matdes.2019.107830.
- 608 [16] T. G. Zieliński, K. C. Opiela, P. Pawłowski, N. Dauchez, T. Boutin, J. Kennedy, D. Trimble, H. Rice, B. Van Damme, G. Hannema, R. Wróbel,
609 S. Kim, S. Ghaffari Mosanenzadeh, N. X. Fang, J. Yang, B. Briere de La Hossieraye, M. C. Hornikx, E. Salze, M.-A. Galland, R. Boonen,
610 A. Carvalho de Sousa, E. Deckers, M. Gaborit, J.-P. Groby, Reproducibility of sound-absorbing periodic porous materials using additive
611 manufacturing technologies: Round robin study, Additive Manufacturing 36 (2020) 101564. doi:10.1016/j.addma.2020.101564.
612 URL <https://linkinghub.elsevier.com/retrieve/pii/S2214860420309362>
- 613 [17] D. C. Akiwate, M. D. Date, B. Venkatesham, S. Suryakumar, Acoustic properties of additive manufactured narrow tube periodic structures,
614 Applied Acoustics 136 (2018) 123–131. doi:10.1016/j.apacoust.2018.02.022.
- 615 [18] Z. Liu, J. Zhan, M. Fard, J. L. Davy, Acoustic properties of a porous polycarbonate material produced by additive manufacturing, Materials
616 Letters 181 (2016) 296–299. doi:10.1016/j.matlet.2016.06.045.
- 617 [19] L. Barguet, V. Romero-García, N. Jiménez, L. M. Garcia-Raffi, V. J. Sánchez-Morcillo, J.-P. Groby, Natural sonic crystal absorber constituted
618 of seagrass (*posidonia oceanica*) fibrous spheres 11 (1) 711. doi:10.1038/s41598-020-79982-9.
- 619 [20] M. R. Stinson, The propagation of plane sound waves in narrow and wide circular tubes, and generalization to uniform tubes of arbitrary
620 cross-sectional shape, The Journal of the Acoustical Society of America 89 (2) (1991) 550–558. doi:10.1121/1.400379.
- 621 [21] D. Lafarge, P. Lemarinié, J. Allard, V. Tarnow, Dynamic compressibility of air in porous structures at audible frequencies, J. Acoust. Soc.
622 Am. 102 (1997) 1995–2006. doi:10.1121/1.419690.
- 623 [22] J.-F. Allard, N. Atalla, Propagation of sound in porous media: modelling sound absorbing materials, 2nd Edition, Wiley, Hoboken, N.J, 2009.
- 624 [23] T. Cavalieri, J. Boulvert, L. Schwan, G. Gabard, V. Romero-García, J.-P. Groby, M. Escoufflaire, J. Mardjono, Acoustic wave propagation in
625 effective graded fully anisotropic fluid layers, The Journal of the Acoustical Society of America 146 (5) (2019) 3400–3408. doi:10.1121/
626 1.5131653.
- 627 [24] V. Dubos, A. Khettabi, D. H. Keefe, C. J. Nederveen, A. Pijnacker, Theory of Sound Propagation in a Duct with a Branched Tube Using
628 Modal Decomposition, acta acustica 85 (1999) 18.
- 629 [25] J.-P. Groby, A. Duclos, O. Dazel, L. Boeckx, W. Lauriks, Absorption of a rigid frame porous layer with periodic circular inclusions backed
630 by a periodic grating, The Journal of the Acoustical Society of America 129 (5) (2011) 3035–3046. doi:10.1121/1.3561664.
- 631 [26] L. Lagarrigue, J.-P. Groby, V. Tournat, O. Dazel, B. Nennig, Acoustic panel.
- 632 [27] T. Cavalieri, J. Boulvert, V. Romero-García, G. Gabard, M. Escoufflaire, J. Regnard, J.-P. Groby, Rapid additive manufacturing of optimized
633 anisotropic metaporous surfaces for broadband absorption 129 (11) (2021) 115102. doi:10.1063/5.0042563.
634 URL <https://aip.scitation.org/doi/10.1063/5.0042563>
- 635 [28] W. Chen, S. Liu, L. Tong, S. Li, Design of multi-layered porous fibrous metals for optimal sound absorption in the low frequency range,
636 Theoretical and Applied Mechanics Letters 6 (1) (2016) 42–48. doi:10.1016/j.tam1.2015.12.002.

- 637 [29] J. Boulvert, T. Cavalieri, J. Costa-Baptista, L. Schwan, V. Romero-García, G. Gabard, E. R. Fotsing, A. Ross, J. Mardjono, J.-P. Groby,
638 Optimally graded porous material for broadband perfect absorption of sound, *Journal of Applied Physics* 126 (17) (2019) 175101. doi :
639 10.1063/1.5119715.
- 640 [30] T. Cavalieri, J. Boulvert, G. Gabard, V. Romero-García, M. Escoufflaire, J. Regnard, J.-P. Groby, Graded and anisotropic porous materials for
641 broadband and angular maximal acoustic absorption 13 (20) 4605. doi : 10.3390/ma13204605.
642 URL <https://www.mdpi.com/1996-1944/13/20/4605>
- 643 [31] B. Howerton, T. Parrott, Validation of an Acoustic Impedance Prediction Model for Skewed Resonators, in: 15th AIAA/CEAS Aeroacoustics
644 Conference (30th AIAA Aeroacoustics Conference), American Institute of Aeronautics and Astronautics, Miami, Florida, 2009. doi :
645 10.2514/6.2009-3143.
- 646 [32] X. Cai, Q. Guo, G. Hu, J. Yang, Ultrathin low-frequency sound absorbing panels based on coplanar spiral tubes or coplanar Helmholtz
647 resonators, *Applied Physics Letters* 105 (12) (2014) 121901. doi : 10.1063/1.4895617.
- 648 [33] J. Boulvert, J. Costa-Baptista, T. Cavalieri, V. Romero-García, G. Gabard, E. R. Fotsing, A. Ross, M. Perna, J. Mardjono, J.-P. Groby,
649 Folded metaporous material for sub-wavelength and broadband perfect sound absorption, *Applied Physics Letters* 117 (25) (2020) 251902.
650 doi : 10.1063/5.0032809.
651 URL <http://aip.scitation.org/doi/10.1063/5.0032809>
- 652 [34] R. Sugimoto, P. Murray, R. J. Astley, Folded cavity liners for turbofan engine intakes, in: 18th AIAA/CEAS Aeroacoustics Conference (33rd
653 AIAA Aeroacoustics Conference), 2012, p. 2291.
- 654 [35] A. T. Chambers, J. M. Manimala, M. G. Jones, Design and optimization of 3d folded-core acoustic liners for enhanced low-frequency
655 performance, *AIAA Journal* 58 (1) (2020) 206–218.
656 URL <https://doi.org/10.2514/1.J058017>
- 657 [36] B. Brouard, D. Lafarge, J.-F. Allard, A general method of modelling sound propagation in layered media 183 (1) 129–142.
- 658 [37] S. W. Rienstra, *Fundamentals of Duct Acoustics* (2015) 52.
659 URL www.win.tue.nl/~sjoerdr/papers/VKI_Rienstra.pdf
- 660 [38] W. Bi, V. Pagneux, D. Lafarge, Y. Aurégan, Modelling of sound propagation in a non-uniform lined duct using a Multi-Modal Propagation
661 Method, *Journal of Sound and Vibration* 289 (4-5) (2006) 1091–1111. doi : 10.1016/j.jsv.2005.03.021.
- 662 [39] J. Guo, X. Zhang, Y. Fang, Broadband forbidden transmission by multiple CPA-based detuned Helmholtz resonators, in: ICSV26, Montréal,
663 2019, p. 8.
- 664 [40] J. A. Nelder, R. Mead, A Simplex Method for Function Minimization, *The Computer Journal* 7 (4) (1965) 308–313. doi : 10.1093/comjnl/
665 7.4.308.
- 666 [41] J. Guo, Reflected wave manipulation and noise attenuation based on inhomogeneous-distributed resonators, Ph.D., The Hong Kong Uni-
667 versity of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, pages: 991012763468803412 (2019). doi : 10.14711/
668 thesis-991012763468803412.
669 URL <http://lbezone.ust.hk/bib/991012763468803412>
- 670 [42] M. E. D’Elia, T. Humbert, Y. Aurégan, Effect of flow on an array of Helmholtz resonators: Is Kevlar a “magic layer”?, *The Journal of the*
671 *Acoustical Society of America* 148 (6) (2020) 3392–3396. doi : 10.1121/10.0002642.
672 URL <http://asa.scitation.org/doi/10.1121/10.0002642>
- 673 [43] J. Boulvert, T. Humbert, V. Romero-Garcia, G. Gabard, E. R. Fotsing, A. Ross, J. Mardjono, J.-P. Groby, Asymmetric metaporous treatment:
674 Optimization for perfect sound absorption, 3d printing, and characterization with air flow, in: 2021 Fifteenth International Congress on Arti-
675 ficial Materials for Novel Wave Phenomena (Metamaterials), IEEE, pp. 065–067. doi : 10.1109/Metamaterials52332.2021.9577098.
- 676 [44] J. Boulvert, J. Costa-Baptista, T. Cavalieri, M. Perna, E. R. Fotsing, V. Romero-García, G. Gabard, A. Ross, J. Mardjono, J.-P. Groby, Acoustic
677 modeling of micro-lattices obtained by additive manufacturing, *Applied Acoustics* 164 (2020) 107244. doi : 10.1016/j.apacoust.2020.
678 107244.